Agilent EEsof EDA

RF and Microwave Oscillator Phase Noise

Agilent Technologies

RIGOROUS RF AND MICROWAVE OSCILLATOR PHASE NOISE CALCULATION BY ENVELOPE TRANSIENT TECHNIQUE

E. Ngoya, J. Rousset, and D. Argollo

IRCOM –UMR CNRS 6615– University of Limoges– 123, Av. A.Thomas, 87060 Limoges Cédex (France)
ngoya@ircom.unilim.fr

ABSTRACT

A novel method is presented which allows accurate calculation of RF and microwave oscillator phase noise. A thorough frequency domain amplitude and phase noise equation is derived for arbitrary circuit topology. The method is based on envelope transient simulation technique. It also speeds up analysis of oscillator transients and circuits driven by chirp and frequency modulated signals.

INTRODUCTION

Oscillator phase noise characterization is of prime importance in RF and microwave communication systems design. If rigorous and general purpose CAD techniques for noise calculation in forced circuits like mixers and amplifiers are well established today [1-2], this is not the case for free running oscillators. It has been observed by several authors that the classical conversion matrix approach used to compute noise in forced circuits is not sufficient for computing phase noise of free running oscillator because the system is ill-conditioned. Important works has been done by authors in last years with the aim of resolving this problem, through both time domain integration (TDI) and harmonic balance (HB) principles[3-6]. Time domain techniques unfortunately have the inconvenient of being hardly applicable to distributed element circuits and high Q oscillators. On the other hand solutions carried in frequency domain (FD) around HB technique suffer from its inability to handle oscillator frequency modulation (FM) induced by noise perturbation. For this reason, in [5] for example, noise equilibrium equation has been separated in two non related terms, namely conversion equation and modulation equation, which in reality are intimately linked. In [6] an alternate technique is presented which claims simultaneous computation of conversion and modulation noise effects, unfortunately the equilibrium equation is likely to be singular as the system matrix involves terms in $1/V_k$, where V_k are circuit node voltage harmonics of the noiseless steady state.

In this paper, a rigorous approach based on Envelope transient (ET) [7] analysis technique is presented. Because ET handles in a natural way oscillator transients in the form of time varying phasors (amplitude, phase and frequency), oscillator phase noise can be readily computed by considering noise signals just as any other modulation signal. We have a direct access to noise induced frequency jitter, amplitude and phase modulation in time domain, from which the overall output noise spectrum is computed by Fourier transform. In the following we are presenting ET equation for analyzing circuits driven by FM carrier signals and oscillator transients, which has not been yet considered in the literature. Based on this a straightforward derivation of oscillator (free running or forced) phase noise analysis is done. A simulation of MESFET oscillator is presented with a comparison against the conventional conversion matrix approach showing the effectiveness of the new method.

COMPUTING RESPONSE OF CIRCUITS DRIVEN BY FM CARRIERS

The commonly adopted form of RF and microwave circuit equation in frequency domain is:

$$\begin{cases} \boldsymbol{A}(\omega)\boldsymbol{X}(\omega)+\boldsymbol{B}(\omega)\boldsymbol{Y}(\omega)+\boldsymbol{D}(\omega)\boldsymbol{G}(\omega) = 0 \\ -\omega_{\max} < \omega < \omega_{\max} \\ \boldsymbol{y}(\boldsymbol{t})=\psi(\boldsymbol{x}(\boldsymbol{t})) \end{cases} \quad (1)$$

where $\boldsymbol{G}(\omega)$, $\boldsymbol{Y}(\omega)$ and $\boldsymbol{X}(\omega)$ are respectively Fourier coefficient vectors of the driving sources $\boldsymbol{g}(\boldsymbol{t})$, nonlinear sources $\boldsymbol{y}(\boldsymbol{t})$ and state variables of the circuit $\boldsymbol{x}(\boldsymbol{t})$. $\psi(\boldsymbol{x}(\boldsymbol{t}))$ is a static characteristic defining the nonlinear sources. In the most general case $\boldsymbol{x}(\boldsymbol{t})$ contains all node voltages and non admittance element currents, plus their derivatives and time shifted versions:

$$\boldsymbol{x}(\boldsymbol{t})=[\boldsymbol{v}(\boldsymbol{t}),\boldsymbol{i_z}(\boldsymbol{t}),\frac{\boldsymbol{dv}}{\boldsymbol{dt}},\frac{\boldsymbol{di_z}}{\boldsymbol{dt}},\boldsymbol{v}(\boldsymbol{t}-\tau_{\boldsymbol{v}}),\boldsymbol{i_z}(\boldsymbol{t}-\tau_{\boldsymbol{z}})].$$

For the following, let us consider that any electrical variable $\boldsymbol{w}(\boldsymbol{t})$ can be decomposed as a sum of finite bandwidth modulations around harmonically related carriers $\omega_{\boldsymbol{k}}$ as below:

$$\boldsymbol{w}(\boldsymbol{t})=\sum_{\mathrm{k}=-\boldsymbol{N}}^{\boldsymbol{N}}\widehat{\mathrm{W}}_{\mathrm{k}}(\boldsymbol{t})\boldsymbol{e}^{\mathrm{j}\omega_{\boldsymbol{k}}\boldsymbol{t}} \quad (2)$$

$$\widehat{\mathrm{W}}_{\mathrm{k}}(\boldsymbol{t})=\frac{1}{2\pi}\int_{-\boldsymbol{BW}/2}^{\boldsymbol{BW}/2}\widehat{\boldsymbol{W}}(\Omega)\boldsymbol{e}^{\mathrm{j}\Omega\boldsymbol{t}}\boldsymbol{d}\Omega$$

$$\omega_{\mathrm{k}}=\boldsymbol{K}^{\boldsymbol{T}}\Lambda$$

Hence under the assumption that the transfer functions $\boldsymbol{A}(\omega)$, $\boldsymbol{B}(\omega)$ and $\boldsymbol{D}(\omega)$ can be closely approximated by a first order polynomial around all ω_{k}, within the modulation bandwidth $\boldsymbol{BW}$, we can derive a first order differential equation [7-8]↑ from (1), which describes the dynamics of time varying phasors (or modulations) of the various variables.

↑ *The above assumption is always true for all lumped element circuits, using modified nodal formulation and is valid for practical modulation bandwidths as regards distributed elements*

$$A(\omega_k)\widehat{X}_k(t) + \dot{A}_k d\widehat{X}_k(t)/dt + B(\omega_k)\widehat{Y}_k(t) + \dot{B}_k d\widehat{Y}_k(t)/dt + D(\omega_k)\widehat{G}_k(t) + \dot{D}_k d\widehat{G}_k(t)/dt = 0 \quad (4)$$

$$\widehat{Y}_k(t) = F_{\tau_c}[\psi(\sum_{p=-H}^{H} \widehat{X}_p(t) e^{j\omega_p \tau_c})] \otimes \delta(f - f_k)$$

$$k = -H,\ldots,H$$

In the above equation $\dot{A}_k, \dot{B}_k, \dot{D}_k$ stand for derivatives of $A(\omega), B(\omega), D(\omega)$ at frequency ω_k and $F_{\tau c}[]$ for Fourier transform along carrier time axis τ_c

Equation (4) can be solved using trapezoidal or Gear integration rule from time origin to a desired time point or until the transient dies out. At this point it is important to notice that signals are sampled at the rate of the time varying complex envelopes (amplitude and phase), not the carrier waveform. This is why ET is superior to conventional brute force TDI. Nevertheless in the case the modulation contains an important frequency modulation index, the resulting amplitude and phase can have a large rate as compared to modulation duration or period, and this still necessitates a large number of envelope samples.

To resolve this problem, it is necessary to keep track explicitly of any frequency modulation term present in the driving signal. For simplicity of the notation, we will hereafter consider only the case of a single fundamental carrier ω_0. The derivations can be easily extended to the multiple fundamental carrier case.

Let us consider a driving signal $g(t)$ with a time varying carrier frequency term (frequency modulation) $\delta\omega_0(t)$ and complex envelope $\widehat{G}_1(t)$:

$$g(t) = \Re e[\widehat{G}_1(t) e^{j\omega_0 t + \int_0^t \delta\omega_0(\tau) d\tau}] \quad (4)$$

When applied to the nonlinear circuit all variables defined by eq (2) now take the form:

$$w(t) = \sum_{k=-H}^{H} \widetilde{W}_k(t) e^{jk\omega_0 t} \quad (5)$$

where $\widetilde{W}_k(t) = \widehat{W}_k(t) e^{jk\int_0^t \delta\omega_0(\tau) d\tau}$

Expressed in this form we can sample separately the AM&PM modulation term $\widehat{W}_k(t)$ and the FM term $\delta\omega_0(t)$ instead of the total compound modulation term $\widetilde{W}_k(t)$ which has a much higher rate. Reconsidering (4) with signal expression defined by (5) we readily find the following equation.

$$A(k\omega_0 + k\delta\omega_0(t))\widehat{X}_k(t) + \dot{A}_k d\widehat{X}_k(t)/dt + B(k\omega_0 + k\delta\omega_0(t))\widehat{Y}_k(t) + \dot{B}_k d\widehat{Y}_k(t)/dt + D(k\omega_0 + k\delta\omega_0(t))\widehat{G}_k(t) + \dot{D}_k d\widehat{G}_k(t)/dt = 0 \quad (6)$$

$$k = -H,\ldots,H$$

Eq (6) is very suitable for all systems driven by chirp type signals like radars and especially for oscillator transient and phase noise simulation as it will be shown.

OSCILLATOR TRANSIENTS SIMULATION

During transient states, oscillator frequency and amplitude are moving continuously so that its dynamics is well described by eq (6), except that the frequency modulation term $\delta\omega_0(t)$ is also an unknown of the system. An elegant way to adress this problem is to use an oscillator probe [9]. An oscillator probe is an active voltmeter whose equivalent circuit is shown in Fig.1.

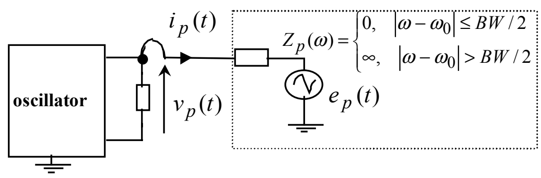

Fig.1 *Oscillator probe*

The probe contains an ideal voltage source supplying a signal of arbitrary amplitude and time varying frequency $e_p(t) = \Re e[\widehat{E}p_1(t) e^{j\omega_0 t + \int_0^t \delta\omega_0(\tau) d\tau}]$ in series with an ideal filter which has zero impedance in a bandwidth BW around the reference frequency ω_0 and infinite impedance outside. The bandwidth is large enough to pass all the energy spectrum delivered by $e_p(t)$. The aim of the probe is to supply the unknown time varying frequency term $\delta\omega_0(t)$ and to fix a phase reference to oscillator, setting instantaneous phase of probe stimulus to some arbitrary value. When connected to a circuit node, the probe must satisfy the constraint of zero load in order not to disturb the circuit equilibrium, i.e no current flowing into probe:

$$\widehat{I}p_1(t) = \eta_p\left(\left|\widehat{E}p_1(t)\right|, \delta\omega_0(t)\right) = 0 \quad (7)$$

Eq (7) introduces two supplementary equations (real and imaginary of $\widehat{I}p_1(t)$) with one more unknown $\left|\widehat{E}_{p1}(t)\right|$ to be added to (6). Simultaneous solution of (6) and (7) gives the time varying complex amplitude $\widehat{X}_k(t)$ of the state variables along with the time varying frequency term $\delta\omega_0(t)$ of the oscillator. Because we are directly sampling frequency, TDI of equilibrium equation can be carried with even very coarse time steps.

OSCILLATOR PHASE NOISE ANALYSIS

It is today well admitted [10] that the basic random fluctuations (noise sources) taking place in electronic devices can be well described by a summation of distinct sinusoids (pseudo sinusoid) with random amplitude and phase ($|N_i|$, θ_i)

$$n(t) = \sum_i |N_i| e^{j\theta_i} e^{j\Omega_i t} = \sum_i N_i e^{j\Omega_i t} \quad (8)$$

Since energy level of noise can be assumed sufficiently small, superposition theorem holds and response of the circuit as a result of noise perturbation can be computed on a frequency by frequency basis. However in a nonlinear circuit pumped by a quasi-periodic drive at a set of frequency $\{\omega_k, k=-H,...,H\}$, there will be an exchange of energy between noise sinusoids at all frequencies of the set $\Theta_s = \{\omega_k \pm \Omega_s, k=-H,...,H\}$ Consequently it is indispensable to carry analysis simultaneously for all sinusoids at frequencies defined by the set Θ_s. The noise frequency sample Ω_s is called noise sideband frequency

Considering that oscillator circuit described by eq (6)-(7) is perturbed by a spot noise sample

$$n_s(t) = \sum_{k=-H}^{H} \hat{N}_{sk}(t) e^{jk\omega_0 t} \quad (9)$$

$$\hat{N}_{sk}(t) = N_k^+ e^{j\Omega_s t} + N_k^- e^{-j\Omega_s t}$$

we need to add (9) as a new driving source into eq (6)-(7). For this purpose it is necessary to accommodate (9) in the form of a FM signal by writing:

$$\hat{N}_{sk}(t) = \hat{N}_k(t) e^{jk\int_0^t \Delta\omega_0(\tau)d\tau} \quad (10)$$

$$\hat{N}_k(t) = \hat{N}_{sk}(t) e^{-jk\int_0^t \Delta\omega_0(\tau)d\tau}$$

Eq (6) may then be rewritten as below to account for contribution of $n_s(t)$

$$\begin{aligned} &A(k\omega_0 + k\delta\omega_0(t))\hat{X}_k(t) + \dot{A}_k d\hat{X}_k(t)/dt + \quad (11)\\ &B(k\omega_0 + k\delta\omega_0(t))\hat{Y}_k(t) + \dot{B}_k d\hat{Y}_k(t)/dt + \\ &D(k\omega_0 + k\delta\omega_0(t))\hat{G}_k(t) + \dot{D}_k d\hat{G}_k(t)/dt = \\ &\qquad F(k\omega_0 + k\delta\omega_0(t))\hat{N}_k(t) + \dot{F}_k d\hat{N}_k(t)/dt \end{aligned}$$

Suppose that the oscillator has no other RF drive except noise, and that we have chosen reference frequency ω_0 to be exactly the steady state oscillation frequency. In this case when we reach the steady state, $\delta\omega_0(t)$ is identically equal to the frequency jitter induced by noise and the state variable $\hat{X}_k(t)$ writes $\hat{X}_k(t) = \hat{X}_{k0} + \delta\hat{X}_k(t)$ where $\delta\hat{X}_k(t)$ is the noise response and $\hat{X}_{k0}$ the noiseless steady state.

At this stage eq (11) can be solved in a direct TDI approach as outlined before. Alternatively, since $\delta\hat{X}_k(t)$ and $\delta\omega_0(t)$ are small, it is numerically more efficient to derive a noise perturbation equation from (11) by a linear expansion around the noiseless equilibrium. Doing so and eliminating the steady state terms, we get oscillator noise envelope transient equation

$$\begin{aligned} &A_k \delta\hat{X}_k(t) + \dot{A}_k \frac{d\hat{X}_k}{dt} + jk\hat{X}_{k0}\dot{A}_k \delta\omega_0(t) + \quad (12)\\ &B_k \sum_p \frac{\partial \hat{Y}_{k0}}{\partial \hat{X}_{p0}} \delta\hat{X}_k(t) + \dot{B}_k \sum_p \frac{\partial \hat{Y}_{k0}}{\partial \hat{X}_{p0}} \frac{d\hat{X}_k}{dt} + jk\dot{B}_k \hat{Y}_{k0} \delta\omega_0(t) \\ &= F_k \hat{N}_{sk}(t) + \dot{F}_k \frac{d\hat{N}_{sk}(t)}{dt} \end{aligned}$$

Eq (12) is a linear equation that can be efficiently solved in frequency domain. In fact since we are considering a sinusoidal noise perturbation, the noise response and frequency jitter are also sinusoidal so that we may write:

$$\delta\hat{X}_k(t) = \Delta X_k^+ e^{j\Omega_s t} + \Delta X_k^- e^{-j\Omega_s t} \quad (13)$$

$$\delta\omega_0(t) = \Re e[\Delta\omega_0 e^{j\Omega_s t}]$$

Considering (13) into (12) and taking account of (7), we readily find oscillator frequency domain noise equilibrium equation, where we can see that conversion and modulation effects are intimately linked in a single equation of the form.

$$\begin{bmatrix} C(\Omega_s) & M \\ u_p^t & 0 \end{bmatrix} \begin{bmatrix} \Delta\vec{X} \\ \Delta\omega_0 \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix} \vec{N} \quad (14)$$

where

$\Delta\vec{X} = [\Delta X_{-H}^+, ..., \Delta X_H^+]^T$, $\Delta\vec{N} = [\Delta N_{-H}^+, ..., \Delta N_H^+]^T$, $C(\Omega_s)$ is the conversion matrix, M is the frequency modulation gradient and u_p is the extraction vector of the oscillator probe current, stating the oscillator arbitrary phase reference condition $\Delta Vp_1 = Vp_{-1} = \Delta Ep_1$.

Considering the signal definitions (5) and (13) and the assumption that frequency jitter due to noise is small compared to Ω_s the total noise complex amplitude at positive sideband frequency Ω_s, around harmonic k is found to be:[↑]

$$\Delta\tilde{X}_k^+ = \frac{kX_{k0}}{2\Omega_s}\Delta\omega_0 + \Delta X_k^+ \quad (15)$$

From (14) and (15) the complete output noise correlation is fully determined by inspection technique.

Given the three correlation terms $\overline{\Delta\tilde{X}_k^+ (\Delta\tilde{X}_k^+)^*}$, $\overline{\Delta\tilde{X}_{-k}^+ (\Delta\tilde{X}_{-k}^+)^*}$ and $\overline{\Delta\tilde{X}_{-k}^+ (\Delta\tilde{X}_k^+)^*}$ oscillator PM noise at some node voltage is computed by the traditional formula.

$$\overline{\delta\phi_k^2(\Omega_s)} = \frac{\overline{\left|\Delta\tilde{V}_k^+\right|^2} + \overline{\left|\Delta\tilde{V}_{-k}^+\right|^2} - 2\Re e\left\{\overline{\Delta\tilde{V}_{-k}^+ \left(\Delta\tilde{V}_k^+\right)^*} e^{j2\theta_{Vk0}}\right\}}{|V_{k0}|^2}$$

[↑] *Otherwise total noise spectrum needs to be computed by DFT of overall modulation term in eq (5). This occurs as Ω_s tends to zero*

APPLICATION

We have used the above outlined method to compute the output frequency and power of the MESFET oscillator described in reference [9], when it is forced by a chirp pulse starting at 100ns and ending at 900ns. Fig.2 shows the time varying input and output frequencies. We may see that the oscillator gets locked from 400ns to 560ns, that is for input frequency in the range 3 to 3.1GHz.

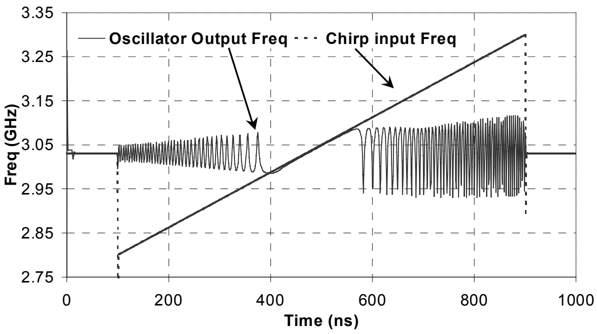

Fig.2 *Time varying oscillator input and output frequency*

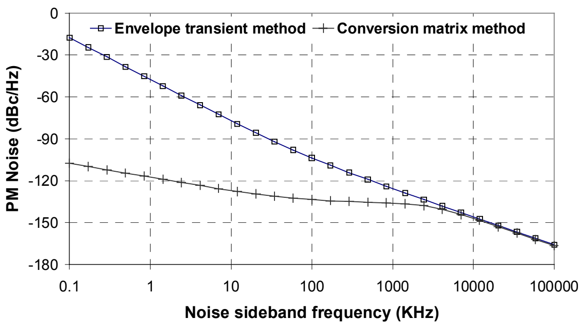

Fig.3 *Oscillator phase noise: comparisonof new method and conversion matrix approach*

Fig.3 shows the simulated phase noise spectrum of the oscillator using the new method and the conventional conversion matrix approach. We can observe important differences between the two simulations enforcing the observation that plain frequency conversion approach can be largely inaccurate for oscillator phase noise analysis. For this simulation MESFET noise was modeled by 1/f flicker current noise, along with channel shot noise. The new method predicts very well the practically observed 30dB/decade slope of phase noise figure near the carrier and naturally coincides to the conversion matrix figure as noise sideband frequency is large.

CONCLUSION

A new method for simulation of systems driven by FM signals and phase noise analysis of arbitrary topology oscillators has been presented. It is shown that frequency conversion and modulation effects taking place in a free running oscillator as a result of noise perturbation are intimately linked within a single equation. A general purpose and self contained oscillator phase noise equation is derived in frequency domain.

REFERENCES

[1] R. Q. Twiss, "Nyquist's and Thevenin's theorems generalized for nonreciprocal linear networks", *J. Appl. Phys*., vol. 26, pp. 599-602, May 1955.

[2] V. Rizzoli, F. Mastri and C. Cecchetti, "Computer aided noise analysis of MESFET and HEMT mixers", *IEEE trans. Microwave Theory Tech* , vol. 37, No.9, pp. 1401-1410, Sept.89

[3] F. X. Kaertner, "Determination of the correlation spectrum of oscillators with low noise"., *IEEE trans. Microwave Theory Tech* , vol. 37, No.1, pp. 90-101, Jan. 1989

[4] V. Rizzoli, F. Mastri and D. Masotti, "General noise analysis of nonlinear microwave circuits by the piecewise harmonic balance technique", *IEEE trans. Microwave Theory Tech* , vol. 42, No.5, pp. 807-819, May 1994

[5] W. Anzill, Marion Fillebock and P. Russer, "Low phase noise design of microwave oscillators", *Int. J. Microwave and Millimeter Wave CAE* , vol. 6, No.1, pp. 5-25, Jan. 1996

[6] F. Farzaneh, E. Mehrshahi, "A novel noise analysis method in microwave oscillators based on harmonic balance", *Proceedings of the 28th European Microwave Conference*, pp. 255-260, 1998

[7] E. Ngoya and R. Larchevèque, "Envelope Transient Analysis: A New Method for the Transient And Steady State Analysis of Microwave Communication Circuits and Systems", *IEEE MTT-S Digest*, pp. 1365-1368, 1996

[8] J. Roychowdhury, "Efficient Methods for Simulating Highly Nonlinear Multi-rate Circuits", *Proc. DAC*, 1997

[9] E. Ngoya, A. Suarez, R. Sommet, and R. Quéré, "Steady state analysis of free or forced oscillators by harmonic balance and stability investigation of periodic and quasi-periodic regimes", *Int. J. Microwave and Millimeter Wave CAE* , vol. 5, No.3, pp. 210-223, March. 1995

[10] P. Penfield, , "Circuit theory of periodically driven nonlinear systems", *Proceedings of the IEEE* , vol. 54, No.2, pp. 266-280, Feb. 1966

For more information about Agilent EEsof EDA, visit:

www.agilent.com/find/eesof

Agilent Email Updates

www.agilent.com/find/emailupdates
Get the latest information on the products and applications you select.

Agilent Direct

www.agilent.com/find/agilentdirect
Quickly choose and use your test equipment solutions with confidence.

www.agilent.com

For more information on Agilent Technologies' products, applications or services, please contact your local Agilent office. The complete list is available at:

www.agilent.com/find/contactus

Americas

Canada	(877) 894-4414
Latin America	305 269 7500
United States	(800) 829-4444

Asia Pacific

Australia	1 800 629 485
China	800 810 0189
Hong Kong	800 938 693
India	1 800 112 929
Japan	0120 (421) 345
Korea	080 769 0800
Malaysia	1 800 888 848
Singapore	1 800 375 8100
Taiwan	0800 047 866
Thailand	1 800 226 008

Europe & Middle East

Austria	0820 87 44 11
Belgium	32 (0) 2 404 93 40
Denmark	45 70 13 15 15
Finland	358 (0) 10 855 2100
France	0825 010 700*
	*0.125 €/minute
Germany	01805 24 6333**
	**0.14 €/minute
Ireland	1890 924 204
Israel	972-3-9288-504/544
Italy	39 02 92 60 8484
Netherlands	31 (0) 20 547 2111
Spain	34 (91) 631 3300
Sweden	0200-88 22 55
Switzerland	0800 80 53 53
United Kingdom	44 (0) 118 9276201

Other European Countries:
www.agilent.com/find/contactus

Revised: March 27, 2008

Printed in USA, June 22, 2005
5989-9445EN

Agilent Technologies