



Agilent EEsof EDA

DC, Linear/Nonlinear AC Stability Analysis using Bifurcation and Nyquist

This document is owned by Agilent Technologies, but is no longer kept current and may contain obsolete or inaccurate references. We regret any inconvenience this may cause. For the latest information on Agilent's line of EEsof electronic design automation (EDA) products and services, please go to:

www.agilent.com/find/eesof



DC, LINEAR AC AND NONLINEAR AC STABILITY ANALYSIS USING BIFURCATION AND NYQUIST THEORY

JAKE GOLDSTEIN AND MEHDI SOLTAN



Xpedion Design Systems, Inc.
4677 Old Ironsides Drive
Santa Clara, CA 95054
Tel: 1-877-xpedion
e-mail: info@xpedion.com
www.xpedion.com



DC, LINEAR AC AND NONLINEAR AC STABILITY ANALYSIS USING BIFURCATION AND NYQUIST THEORY

Designing power and low noise amplifiers, gain blocks, multipliers and oscillators for modern communication systems requires delicate trade-offs between various design specifications including linearity, power efficiency, crosstalk between channels and stable operation over a wide range of input power and impedance. Most of the techniques used in amplifier designs, whether for power amplifier linearization or to minimize crosstalk between channels or noise figure, result in complex architectures with several nonlinear or active devices in the circuit. One of the primary criteria for any successful amplifier, multiplier or oscillator design is the assurance of its stable operation at DC and power on and at various input and output signal levels and terminating impedances.

CLASSICAL STABILITY ANALYSIS AND ITS LIMITATIONS

The stability analysis of any RF and microwave circuit is the figure of merit for its consistent operation behavior (for example, no oscillations for an amplifier, or unconditionally sustainable oscillations for an oscillator). Circuit stability can be determined from the S parameters of the active device, input and output matching circuits, and terminations. A

two-port network is said to be unconditionally stable at a given frequency if it is stable against oscillation for all passive source and load impedances. If a two-port network is not unconditionally stable, it is potentially unstable and is said to be conditionally stable. In the second case, some passive source and load impedance combinations can cause the two-port network to oscillate.

For a two-port network, necessary and sufficient conditions for unconditional stability are that the stability factor K (Rollet's factor) is greater than one and the real part of the input immittance parameter (γ_{11} and γ_{22}) is greater than zero, with an overriding condition that the poles of the two-port network under investigation with ideal terminations (open and short circuit) must lie in the left half plane. These criteria are expressed in terms of K and Δ (scattering matrix determinant), and zeros of the characteristic frequencies of the network. In particular, the network is unconditionally stable if $K > 1$ and $|\Delta| < 1$, and the zeros of the characteristic frequencies lie in the left half plane.

JAKE GOLDSTEIN AND MEHDI SOLTAN
Xpedion Design Systems Inc.
Santa Clara, CA

In terms of reflection coefficients and S parameters of the two-port network, the conditions for unconditional stability at a given frequency are

$$|\Gamma_s| < 1, |\Gamma_l| < 1 \text{ and } |\Gamma_{out}| < 1$$

where

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_l}{1 - S_{22}\Gamma_l}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

For, $|\Gamma_s| < 1$ and $|\Gamma_l| < 1$, the necessary and sufficient conditions for unconditional stability are

$$K = \frac{1 - |S_{11}| |S_{11}| - |S_{22}| |S_{22}| + |\Delta| |\Delta|}{2|S_{12}S_{21}|} > 1$$

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1$$

In the conditionally stable case ($K < 1$), the loci of Γ_s and Γ_l in the Smith chart, shown in **Figure 1** where values of Γ_s and Γ_l produce $|\Gamma_{out}| = 1$ and $|\Gamma_{in}| = 1$, are called the input and output stability circles, respectively. The radii r and centers c of the stability circles in the Γ_s and Γ_l planes, respectively, are given as

input stability circle:

$$r_s = \left| \frac{|S_{12}S_{21}|}{|S_{11}| |S_{11}| - |\Delta| |\Delta|} \right|$$

$$c_s = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}| |S_{11}| - |\Delta| |\Delta|}$$

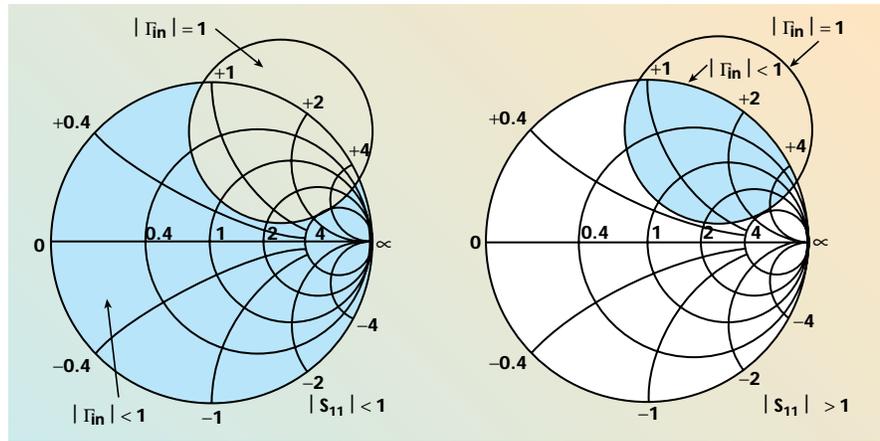
output stability circle:

$$r_L = \left| \frac{|S_{12}S_{21}|}{|S_{22}| |S_{22}| - |\Delta| |\Delta|} \right|$$

$$c_L = \frac{(S_{22} - \Delta S_{11}^*)}{|S_{22}| |S_{22}| - |\Delta| |\Delta|}$$

If $|S_{11}| < 1$, the inner area of the Smith chart, which is outside the output stability circle, represents the stable region. On the other hand, if $|S_{11}| > 0$, the intersection of the inner areas of the Smith chart and the output stability circle represents the stable region. The shaded area represents the stable regions in the Γ_l plane.

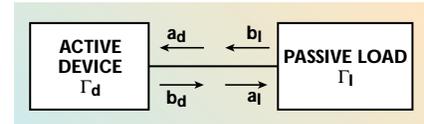
However, there are serious limitations in this widely used process. The



▲ Fig. 1 The output stability circle in the Γ_l plane.

flaw is that stability conditions based on the K factor are strictly valid only for a single-transistor amplifier. When the amplifier contains more than one active device, the K factor test may lead to incorrect conclusions. This scenario is commonly encountered by designers. The method described previously is also insufficient and inaccurate to apply to the stability analysis of nonlinear RF and microwave circuits. The unconditional stability analysis of two-port RF and microwave circuits that is generally performed by designers and proposed in many textbooks and commercial design software packages utilizes only K and Δ parameters. This approach fails in many cases, such as in the analysis of multistage amplifiers or complex networks. Moreover, in practical cases, the S parameters of the active devices do not satisfy the unconditional stability criterion related to K and Δ for all frequencies. In this situation the designer must verify the conditional stability of the network.

In some cases, it is better to perform an analysis for instability, that is, assume the circuit to be an oscillator. The oscillation condition states that the product of the reflection coefficients of the active device and passive load must be equal to 0 ($\phi_d + \phi_l = 0$). It is intuitively clear that in this situation a signal incidentally present on the system is amplified until oscillations are established. Nevertheless, this approach is not always correct because to check the conditional stability it is necessary to know the behavior of the circuit (not only at the resonant frequency, but also for any frequency). Moreover, the result of this method depends on the choice of



▲ Fig. 2 An active RF/microwave network representation.

the normalizing impedance of the reflection coefficient.

An accurate method for checking the stability of a circuit is to consider the Nyquist criterion, which overcomes the limitations of the traditional methods described previously. The Nyquist criterion is a graphical method that allows determination of the stability of a closed-loop system, starting from the system's open-loop transfer function. However, applying the Nyquist criterion requires complex circuit analysis and exact knowledge of the open-loop transfer function of the system under investigation.

STABILITY ANALYSIS USING NYQUIST AND BIFURCATION ANALYSIS

The Nyquist theory method is applicable to small-signal circuits. In the presence of large-signal RF drive (such as in a power amplifier or oscillator), the device nonlinearities can considerably modify the stability pattern, for example, by causing spurious oscillations or generating subharmonics. This behavior cannot be detected by the Nyquist approach since it requires more sophisticated techniques based on bifurcation theory.

By applying the Nyquist criterion to active RF and microwave circuits as shown in **Figure 2**, if Γ_d is the reflection coefficient seen looking into the active device and $\Gamma_l(f)$ is the reflection coefficient seen looking into

the passive circuit, it is possible to define a transfer function similar to that of a closed-loop system. In fact, it is possible to evaluate the ratio of $a_i(f)$ and $a_d(f)$, resulting in

$$\frac{a_i(f)}{a_d(f)} = \frac{\Gamma_d(f)}{1 - \Gamma_d(f)\Gamma_1(f)}$$

The Nyquist criterion is a graphical method that allows determination of the stability of a closed-loop system from the open-loop transfer function with the frequency varying from zero to infinity. If P_{cl} (greater than or equal to zero) is the number of right half-plane poles of the closed-loop transfer function, P_{op} (greater than or equal to zero) is the number of right half-plane poles of the open-loop transfer function and N_i is the number of clockwise encirclements of the critical point (1, j0) (negative if counterclockwise), the closed-loop system is stable if and only if

$$P_{cl} = P_{op} + N_i = 0$$

and if the open-loop transfer function does not cross the critical point. If the open-loop system is stable ($P_{op} = 0$), it is sufficient that $N_i = 0$ (reduced criterion).

Linear Stability Analysis

To illustrate generalized Nyquist stability analysis, the circuit schematic is represented in the classical form of an amplifier with feedback, as shown in **Figure 3**. For this analysis, the circuit elements are separated into two blocks: all active elements in the equivalent amplifier block and all passive elements in the feedback block. It is well known that the circuit is unstable if the feedback is additive. More precisely, the circuit is unstable if the determinant of the equilibrium equation has a zero with a positive real part.

$$\begin{aligned} V_g &= \beta(\rho + j\omega) I_d + \alpha(\rho + j\omega) V_g \\ I_d &= A V_g [1 - \beta(\rho + j\omega) A] V_g \\ &= \alpha(\rho + j\omega) V_g \\ \delta(\omega) &= \det [1 - \beta(j\omega) A] \end{aligned}$$

In practice, it is not possible to compute zeros of the determinant. For diagnosis of the stability, the GoldenGate™ simulator employed for analyses in this article uses Nyquist criteria, that is, the locus of

the determinant with varying frequency is plotted in the complex plane. The circuit is unstable if the determinant locus encircles the origin clockwise. The amount of encircling gives the number of instability frequencies. Intersections of the locus with the negative real axis provide a good estimate of instability frequency (starting oscillation frequencies).

To further simplify interpretation of the Nyquist stability plot, the simulator provides a projection of the natural locus into the phase plane (called unwrapped locus), where the user can easily check the encircling of the origin. The frequency step along the locus is also automatically monitored by the simulator to adapt to the fast or slow variation of the locus. This analysis is extremely important for high Q circuits.

Nonlinear AC Stability Analysis

The principle of stability analysis using harmonic balance is a generalization of the Nyquist principle already seen in the linear analyzer (DC bias stability). It requires introducing a small evanescent perturbation $\rho + j(\omega_k + \omega)$ on all active nodes of the circuit and observing if the resulting circuit output disappears with time. The perturbation equation is obtained by linearizing the harmonic balance equation around the steady-state equilibrium condition.

The resulting equations may be put in the form of amplifier gain with feedback. The open-loop gain of the amplifier is represented by derivatives of the nonlinear sources and the feedback gain is represented by the immittance of the passive subcircuit, as shown in **Figure 4**. The perturbation equilibrium equations are

$$\begin{aligned} I[\omega_k + (\omega - j\rho)] &= \\ Y1[\omega_k + (\omega - j\rho)]V[\omega_k + (\omega - j\rho)] &+ \\ Y2[\omega_k + (\omega - j\rho)]E[\omega_k + (\omega - j\rho)] & \end{aligned}$$

where

$$\omega_k = \sum_i k_i \Omega I \quad (k = 1, \dots, NH)$$

$$\begin{aligned} I[\omega_k + (\omega - j\rho)] &= \\ \left[\frac{\delta I}{\delta V} \right] V[\omega_k + (\omega - j\rho)] & \\ \delta(\omega) &= \det [1 - \beta(j\omega) A] \end{aligned}$$

Following the Nyquist criterion, the circuit is considered unstable if the locus of the determinant of the characteristic equation encircles the origin clockwise when the perturbation frequency is swept from zero to infinity. In practice it is only necessary to sweep the frequency from zero to the maximum oscillation frequency of the active devices. While using the simulator, the user need not determine the frequency step and maximum frequency (except for DC); these parameters are automatically monitored by the simulator. The simulator also detects the encircling or not of the origin. A stability flag, set to one if the circuit is stable and zero if not, is given as an output. The simulator also provides the instability frequencies when the circuit is unstable.

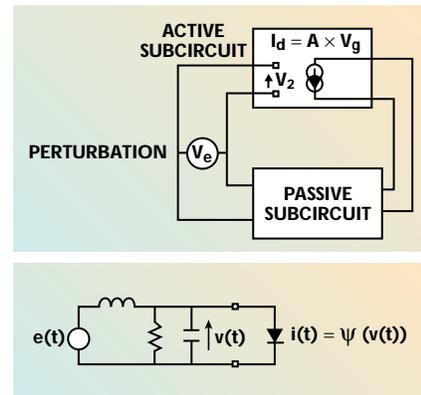
The weakest point of the stability analysis using Nyquist criterion is that it can be computationally intensive, especially when computation of stability margins relating to design parameters is desired. However, when it has already been verified that the bias point is stable, the main risk of instability in large-signal drive conditions is frequency division by two and hysteresis. These two types of instability may be detected with a reasonable computation cost during steady-state simulation. Bifurcation analysis is very useful for computing the locking range of analog frequency dividers and locked oscillators, as shown below.

STEP-BY-STEP DC, LINEAR AC AND NONLINEAR AC STABILITY ANALYSIS

DC Operating Point Stability

DC stability analysis is performed to check if the biasing point of the circuit

Fig. 3 The circuit representation in active and passive segments. ▼



▲ Fig. 4 A nonlinear circuit.

under test is stable or not. For nonautonomous circuits (such as an amplifier, mixer or active filter), DC stability analysis determines that the circuit does not turn into an oscillator as the power supply is turned on. The DC operating point must be stable. Similarly, for autonomous circuits (such as in an oscillator design) DC stability analysis verifies that the circuit actually oscillates when the power supply is turned on. Hence, the DC operating point must be unstable in this case.

DC Stability Analysis

For nonautonomous circuits (amplifiers), DC analysis is performed to locate the biasing point and DC stability analysis is used to determine if the biasing point is stable. If the biasing point is stable, the analysis determines the resonant frequency characteristics of the circuit (stopband or passband frequencies); if the biasing point is unstable, it determines the potential oscillation frequencies.

For autonomous circuits (oscillators), a DC analysis is similarly performed to locate the biasing point. A DC stability analysis is then performed to determine if the biasing point is actually unstable so that oscillations will build up. If the biasing point is unstable, the simulator provides the potential oscillation frequencies; if the biasing point is stable, it provides the resonant frequency characteristics of the circuit for information (stopband or passband frequencies).

Note that in the case of high Q circuits, if the Nyquist plot definition is set up with too small a number, the stability analysis may lead to incorrect conclusions. The simulation results must be carefully analyzed. For example, the simulation results may indicate that the biasing point is stable when it is not. In case of any doubts, it is safe to set the Nyquist plot definition to the highest level. In addition, the only reliable way to determine the startup of oscillation frequency is through DC stability analysis. Nonlinear AC stability analysis should not be used for this purpose.

Harmonic Balance Analysis

In the case of nonautonomous circuits (such as amplifiers), if the DC bias point is stable, AC analysis of the amplifier should be performed (that is, linear AC, S parameter or harmon-

ic balance). Note that if the DC bias is unstable and an AC analysis is performed, misleading results may occur and most often the harmonic balance analysis is terminated with a message of harmonic balance overflow. In the case of autonomous circuits (such as oscillators), if the DC bias point is unstable, AC analysis (that is, harmonic balance analysis) of the oscillator is performed. The stability analysis provides an accurate estimation of the oscillation frequency.

If the circuit is unstable and the DC stability indicates that the bias point is unstable, even when using the highest Nyquist plot definition, harmonic balance oscillator analysis still can be performed with one of the resonant frequencies provided by the stability analysis. If sustainable and stable oscillations are found, the stability of oscillation still needs to be checked to determine if the oscillator is not going to start from DC but will require a starting RF pulse. This requirement may be confirmed by performing a nonlinear stability analysis by analyzing the oscillator as an amplifier.

Large-signal Stability Analysis

Large-signal operating point stability analysis is performed to check if the steady state obtained for an amplifier, mixer or oscillator is maintained long enough to be physically sustained. For nonautonomous circuits, nonlinear stability analysis indicates whether the amplifier continues to be an amplifier for the given input drive and frequency. Sometimes for certain drive levels and input frequencies the amplifier may turn into an oscillator (self-oscillating mixer) or, more often, a frequency divider. For autonomous circuits, nonlinear stability analysis indicates whether oscillations are sustained at this frequency and power. If the stability test indicates an unstable condition, it means that the circuit is actually oscillating at some other frequency, or possibly it has more than one simultaneous oscillation frequency. The actual frequency of oscillation is usually listed in the DC stability analysis results.

Note that during large-signal operating point stability analysis the maximum perturbation frequency must be set to the fundamental frequency tone (in the case of one-tone analysis). The Nyquist plot is nearly

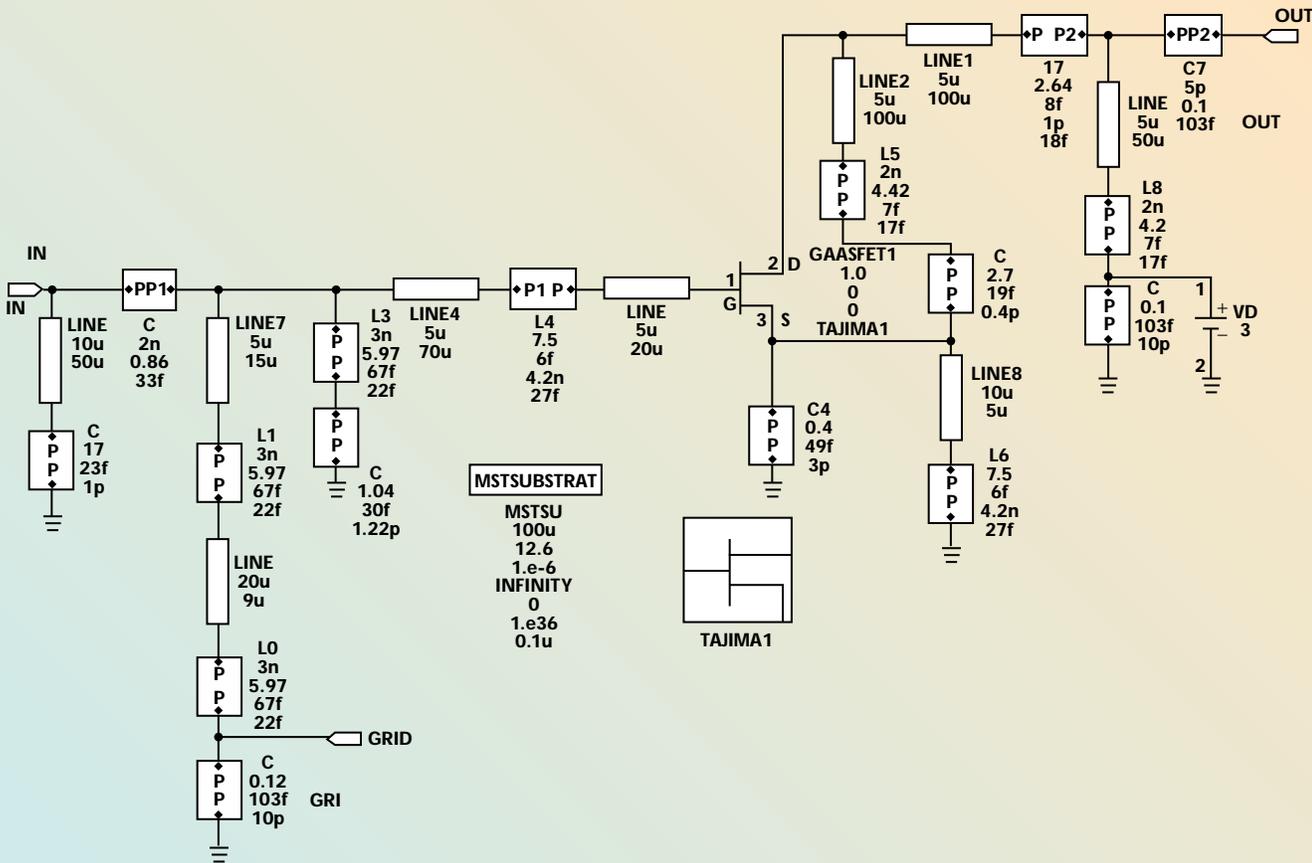
periodic with a period equal to the fundamental tone. The instability frequency provided by the analysis corresponds to an offset frequency from one of the harmonics (DC, $\pm f_0$, $\pm 2f_0$, ..., $\pm Nf_0$), so that the true frequency of instability is actually $|kf_0 \pm f_{\text{ins}}|$.

A STEP-BY-STEP AMPLIFIER AND MULTIPLIER STABILITY ANALYSIS EXAMPLE

A step-by-step process for stability analysis of an amplifier, using the simulator, begins by performing a DC stability analysis to determine if the circuit is stable at the bias point. While setting up this analysis, the stability analysis must be run while the maximum perturbation frequency is set large enough to cover the maximum frequency of oscillation of the device. The maximum frequency may depend on the device and substrate technology or the cutoff frequency of the transistors used in the circuit design.

In the first case, if the DC stability analysis indicates that the circuit is unstable, it also provides the instability frequency or frequencies. The next step is to reconsider the circuit topology or change the active device or devices used in the circuit design until the stability analysis indicates stable DC performance. When the circuit is composed of several active devices, it is interesting to know which devices are contributing to the instability. In the simulator, this detection is simple and very user friendly. The simulator allows the designer to individually turn the target device(s) to a passive state simply by setting the TURNOFF flag to 1 and watching if the instability vanishes. If turning a device to the off state produces stable operation, it indicates that this particular device is contributing to unstable DC operation.

In the second case, if the DC stability analysis indicates that the circuit is stable, the next step is to perform nonlinear harmonic balance analysis to compute the nonlinear steady-state condition of the circuit and perform nonlinear stability analysis to check if the steady-state condition is physically sustained/stable or just an artifact of the mathematical solution of the circuit equations. This step is one of the most important and critical aspects of any circuit design and is ignored by most other commercial RF and microwave simulation tools.



▲ Fig. 5 A 6 GHz amplifier design.

If none of the nonlinear regions of operation is sustainable (that is, unstable), the simulator provides the instability frequency (offset frequency). If f_{inst} is the simulated offset instability frequency, the real frequency of instability is one of the frequencies $f_k \pm f_{inst}$, where f_k is the DC and harmonic (or intermodulation) products of the steady-state region of operation. Usually, it is DC $\pm f_{inst}$ or fundamental tone $\pm f_{inst}$.

The critical information from the nonlinear stability analysis is to determine if the circuit is stable or not (for a given biasing and input drive) or, in other words, if the waveform and power given by the harmonic balance simulator are a real representation of a physical situation. The actual value(s) of the instability frequency(ies) is only secondary information and mostly for informational purposes.

A 6 GHz, GaAs MESFET-based MMIC amplifier design, shown in **Figure 5**, is designed to operate as a frequency divide by two. When biased at -1.8 V gate voltage, this circuit behaves as a times-two frequency multiplier at low and medium input

levels. However, when biased at a -1.8 V gate voltage and the input signal is driven higher than 10 dBm, the amplifier becomes unstable and starts to function as a frequency divide-by-two circuit. This particular behavior can be predicted by nonlinear stability analysis.

DC Stability Analysis

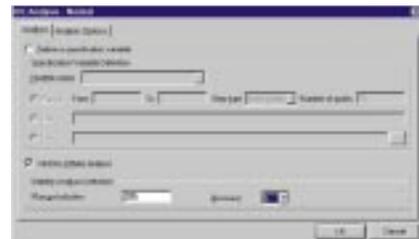
As shown in **Figure 6**, the DC stability analysis is performed with a maximum frequency of 20 GHz (since the circuit is using a GaAs MESFET with a cutoff frequency of 20 GHz), step size of 100 and accuracy level of 3. **Figure 7** shows a Nyquist plot from the DC stability analysis of the amplifier using the simulator. The analysis indicates that the DC bias point of this circuit is stable, which means that this circuit will behave as well as an amplifier under small-signal operation.

Nonlinear Stability Analysis

Nonlinear stability analysis using the simulator's nonlinear harmonic balance is set up as shown in **Figure 8**. The analysis is performed using a

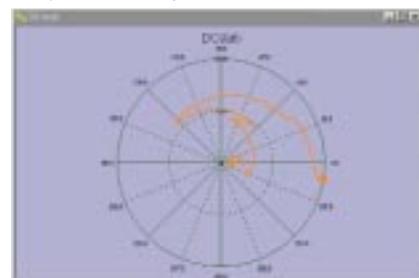
single-tone input frequency with input power levels of 4 and 8 dBm, and considering five harmonics.

As shown in **Figure 9**, the nonlinear stability analysis indicates that the circuit is stable at a 4 dBm input drive, and the waveform and power output indicate that the circuit performs as a



▲ Fig. 6 The DC stability analysis setup.

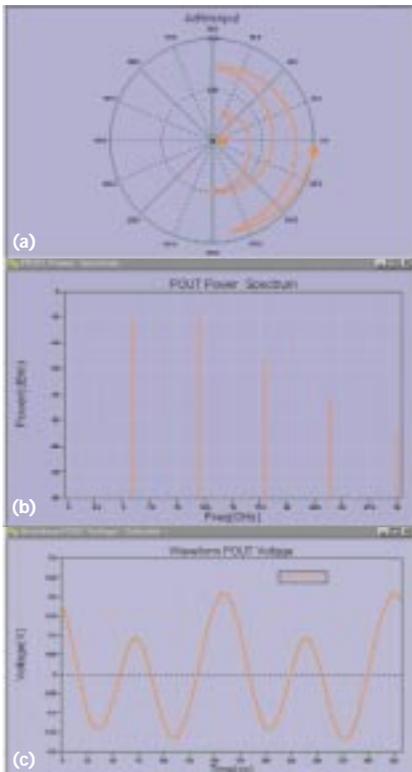
Fig. 7 Nyquist plots from the DC stability analysis results. ▼



multiplier at this input drive level. **Figure 10** shows the harmonic balance results obtained when the nonlinear stability analysis is performed with 8 dBm input drive. The waveform and output power results indicate that the circuit still works as a $2\times$ multiplier with 8 dBm input power. However, when a nonlinear stability analysis of this circuit is performed at 8 dBm input power level, as shown in **Figure 11**, the analysis indicates that the circuit is unstable. Thus, the results from harmonic balance simulation indicate a steady-state



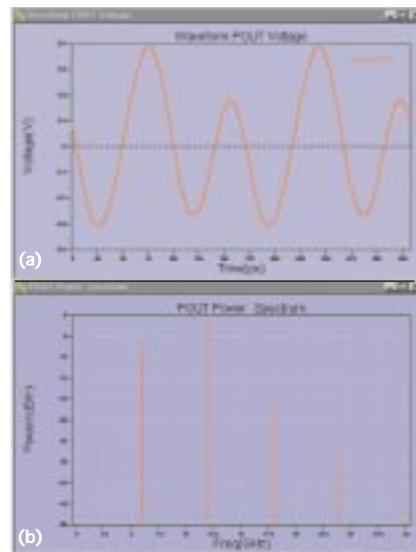
▲ **Fig. 8** The nonlinear stability analysis setup.



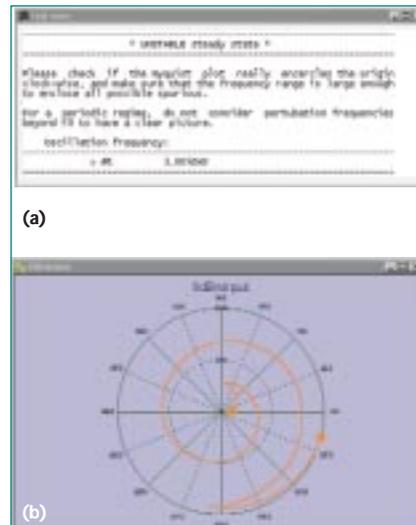
▲ **Fig. 9** Nonlinear analysis results with 4 dBm input drive; the (a) Nyquist plot, (b) output power spectrum and (c) output voltage waveform.

region of nonlinear circuit operation that is not physically sustained; that is, the circuit is nonlinear unstable. **Figure 12** shows the simulator setup screen for nonlinear stability analysis.

The results produced by harmonic balance analysis are only a mathematical solution (or artifact) and not physically realizable. The nonlinear stability analysis shows an instability offset frequency of 3 GHz. As explained previously, the actual instability frequency is $f_0 \pm 3$ GHz, or $DC \pm 3$ GHz, which means it is 3 GHz in this case. Thus, at this input drive level the circuit is operating as a frequency divide-by-two circuit.



▲ **Fig. 10** Nonlinear analysis results with 8 dBm input drive; the (a) output voltage waveform and (b) output power spectrum.



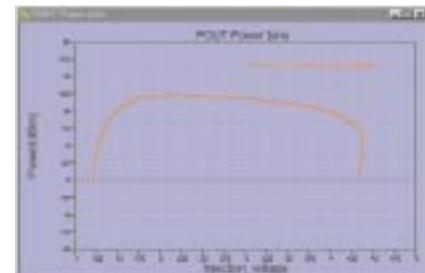
▲ **Fig. 11** The nonlinear stability analysis (a) report and (b) Nyquist plot for 8 dBm input drive.

It is important to note that the waveform and spectrum produced by harmonic balance analysis at an input drive level of 8 dBm look normal for multiplier operation just as for the 4 dBm input. Hence, only looking simply at the output waveform or output power spectrum is quite misleading. It may be impossible to determine, without accurate and extensive nonlinear stability analysis, whether the nonlinear harmonic balance results are just a mathematical artifact of analysis or a physically realizable effect. Accurate nonlinear stability analysis is the only way to determine this fact.

As determined from the above analysis, when the circuit is operating as an oscillator, frequency divide by two, the accurate simulation method is to analyze the circuit in an oscillator mode using nonlinear harmonic balance analysis (since the circuit is unstable, as shown from nonlinear stability analysis). **Figure 13** shows the simulation output plot, which displays the frequency division that starts at approximately 5 dBm (1.2 V) and vanishes at approximately 17 dBm (4.4 V) input power level for a 6 GHz input frequency. A similar analysis is performed, except in this case the input power is fixed at 15



▲ **Fig. 12** The nonlinear stability analysis setup.



▲ **Fig. 13** A locked oscillator power output plot formed by sweeping the input voltage.

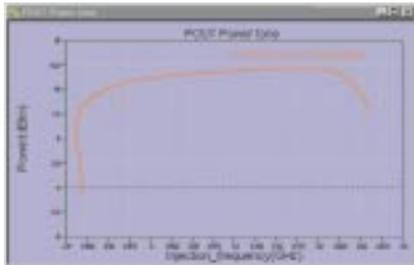
TECHNICAL FEATURE

dBm (3.5 V) and the input frequency is swept from 5 to 8 GHz. The results, shown in **Figure 14**, indicate that at a fixed input power of 15 dBm (3.5 V) the circuit performs as a frequency divider starting at $2 \times 2.91 = 5.82$ GHz and ending at $2 \times 3.259 = 6.518$ GHz.

A STEP-BY-STEP OSCILLATOR STABILITY ANALYSIS EXAMPLE

A step-by-step process for stability analysis of an oscillator using the simulator begins by performing a DC stability analysis to determine the starting frequency(ies) of oscillation(s), if any. These are called instability frequencies (or also, mistakenly, oscillation frequencies).

Fig. 14 A locked oscillator power output plot formed by sweeping frequency with a fixed input voltage. ▼



The stability analysis must be run while the maximum perturbation frequency is set large enough to cover the maximum frequency of oscillation of the device. The maximum frequency may depend on the device and substrate technology or the cutoff frequency of the transistors used in the circuit design.

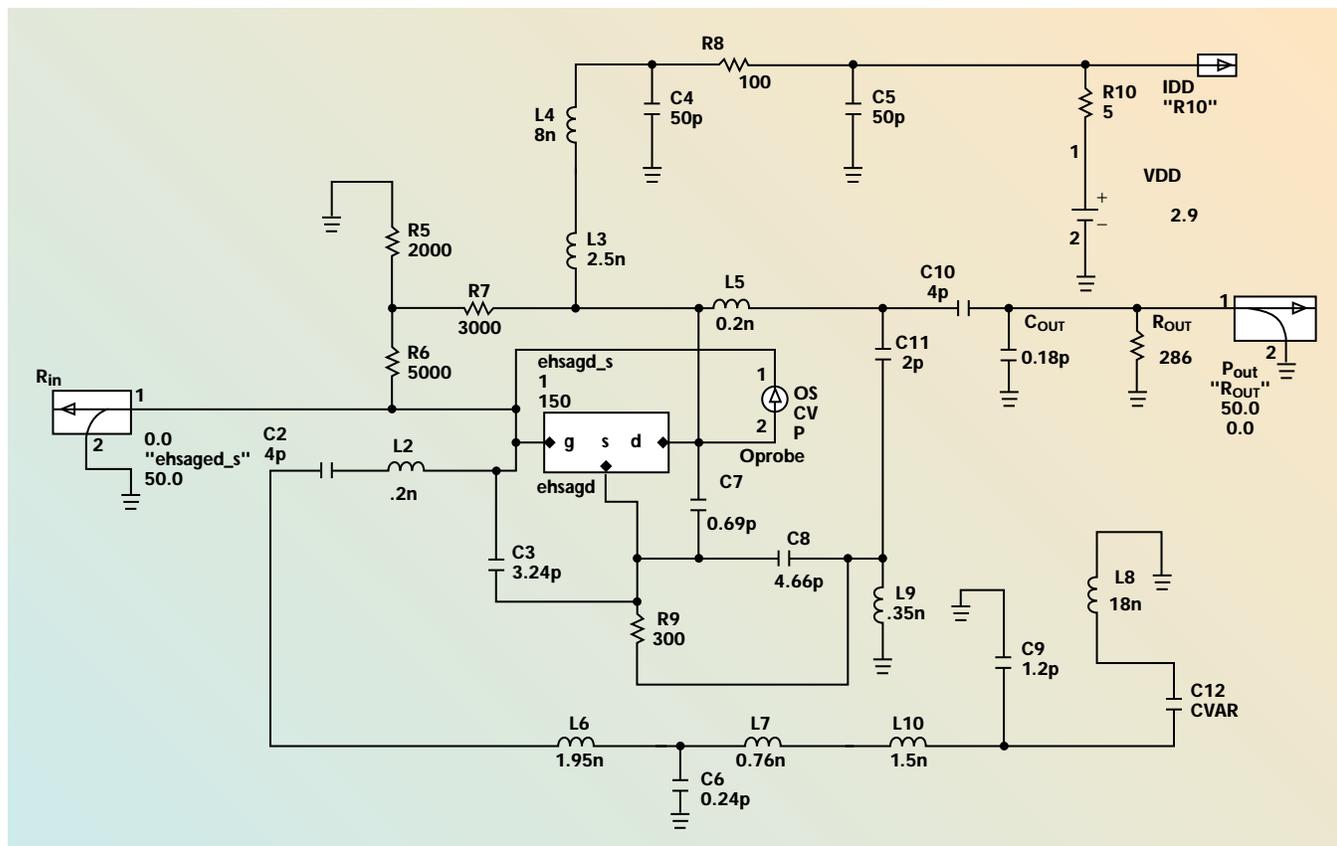
Next, an oscillator analysis is performed to compute the real oscillation frequency and power. This analysis is required only if the DC stability analysis indicates that the circuit is unstable. If the stability analysis results in more than one frequency of instability, an oscillator analysis must be performed for each instability frequency. Therefore, an oscillator analysis with an estimated oscillation frequency equal to every frequency of instability must be run. Only one of these oscillation regimes will be physically sustained.

A nonlinear stability analysis is then performed for each oscillation regime to check if the oscillation regime is physically sustained. If none of the regime is sustained (that is, stable), this means that there is another regime to be found, probably a

two-tone or chaotic regime, in which case the harmonic balance method cannot determine the proper solution. In practice, this condition means that the designer must modify the circuit to have proper oscillation(s). A 0.87 GHz, GaAs MESFET-based MMIC oscillator design example is shown in **Figure 15**.

DC Stability Analysis

The DC stability analysis is performed with the maximum frequency of 10 GHz, step size of 1000 Hz and accuracy level of 3, as shown in **Figure 16**. (Since the circuit is using a GaAs MESFET it was not expected to be unstable at any frequency greater than 10 GHz.) The stability analysis indicates that the DC state of this circuit is unstable, which is acceptable for a circuit designed to be an oscillator. The analysis provides two frequencies of instability (0.87 and 3 GHz), as shown in **Figure 17**. In addition, it can be seen that the Nyquist plot encircles the origin twice. An examination of the Nyquist plot is necessary to verify whether the definition of the diagram is good enough. Good definition means a regular plot with no large

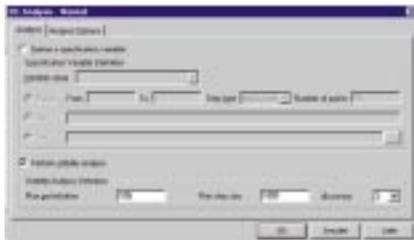


▲ **Fig. 15** A 0.87 GHz oscillator design.

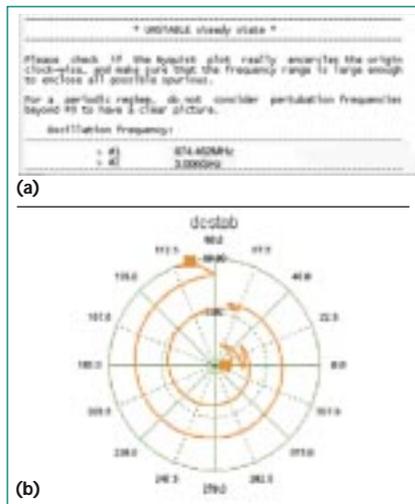
discontinuities of phase. If the definition is not good, the setup must be changed to increase the accuracy parameter and the analysis rerun.

Nominal Oscillator Analysis

The stability analysis indicates that the DC state is unstable (which is expected from this circuit) with two frequencies of instability (possible starting frequencies of oscillation). Therefore, two different oscillator analyses are required to be run, one for each instability frequency. For this analysis the estimated oscillation frequency is set equal to the instability frequency.



▲ Fig. 16 The DC stability analysis setup.



▲ Fig. 17 DC stability analysis (a) report and (b) Nyquist plot.

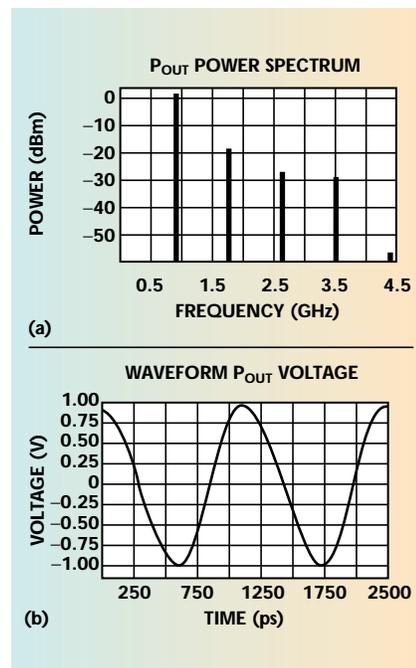


▲ Fig. 18 The nonlinear analysis setup at 0.87 GHz.

First, the oscillation frequency is set equal to the 0.87 GHz instability frequency and five harmonics are considered to account for the nonlinearity of the oscillator, as shown in **Figure 18**. The simulator computes a real oscillation frequency of 881 MHz with the output waveform and power spectrum shown in **Figure 19**. Next, the oscillation frequency is set equal to the 3 GHz instability frequency and five harmonics are considered. The simulator computes a real oscillation frequency of 3 GHz with the output waveform and power spectrum shown in **Figure 20**. The output power is approximately 13 dB lower than for the previous oscillation frequency.

Nonlinear Stability Analysis

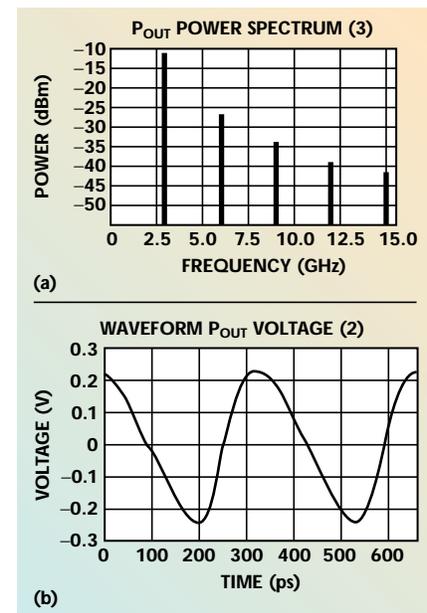
Two oscillation frequencies (mathematical solutions) have been identified, one at 0.88 GHz and the other at 3 GHz. It now must be determined which of the two is really physically sustained, if either. It should be noted that even if only a single mathematical oscillation regime is determined, its stability must be checked to determine if it is really sustained. This is a specific limiting feature of all numerical simulators. The output of a numerical simulator is always a mathematical solution, the physical existence of which must be con-



▲ Fig. 19 Nonlinear analysis (a) output power spectrum and (b) output voltage waveform at 0.87 GHz.

firmed by a stability analysis. In contrast to time domain simulators, harmonic balance, DC and linear frequency domain simulators produce nonphysical solutions because, in the analysis setup, a fixed fundamental frequency basis must be selected. This characteristic is why stability analysis is a very important feature for RF and microwave designs.

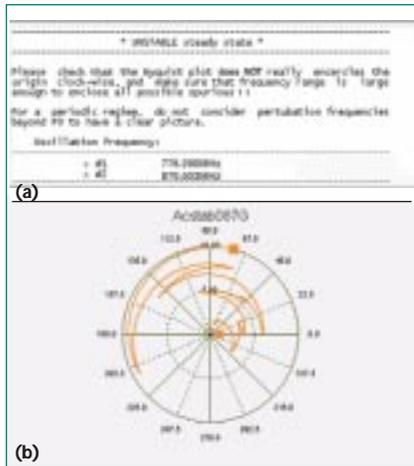
For stability analysis of a periodic signal, such as from the oscillator regime, a maximum perturbation frequency equal to the fundamental frequency of the regime must be set. **Figure 21** shows the setup for 0.88 GHz maximum perturbation, which is the oscillation frequency. The simulator's stability analysis indicates that this oscillation regime will be sus-



▲ Fig. 20 Nonlinear analysis (a) output power spectrum and (b) output voltage waveform at 3 GHz.

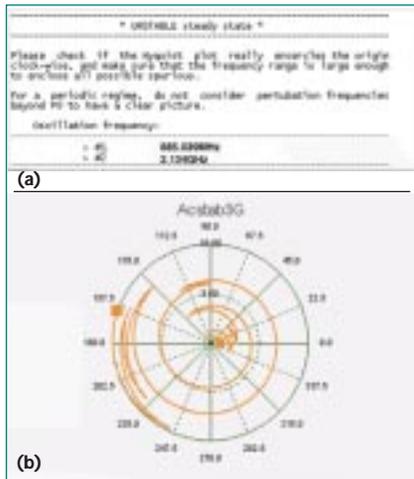
Fig. 21 Nonlinear stability analysis setup for 0.88 GHz oscillation. ▼





▲ Fig. 22 Nonlinear stability analysis (a) results report and (b) Nyquist plot for 0.88 GHz oscillation.

Fig. 23 Nonlinear stability analysis (a) results report and (b) Nyquist plot for 3 GHz maximum perturbation. ▼



tained, as shown in **Figure 22**. Similarly, the maximum perturbation frequency is set equal to the fundamental frequency of 3 GHz. The resulting stability analysis indicates that this regime is unstable, thus it cannot be physically sustained. The analysis also provides the frequencies of instability, as shown in **Figure 23**.

Note that in the case of nonlinear stability, instability frequencies given by the simulator are offset frequen-

cies. If f_{inst} is the offset instability frequency given by the simulator, the real frequency of instability is one of the frequencies $f_k \pm f_{inst}$, where f_k stands for DC and harmonic (or intermodulation) products of the steady-state regime. The simulator cannot solve the indetermination. Usually the determined frequency is DC $+f_{inst}$ or the fundamental tone, $-f_{inst}$, and design knowledge of the circuit leads easily to the proper value.

The main information required for nonlinear stability analysis is whether or not the circuit is stable (for the given biasing and input drive) or, in other words, if the waveform and power provided by the harmonic balance simulator are really physical. The actual value(s) of instability frequency(ies) is only secondary information. The simulator provides this information with some indetermination to be resolved if necessary by the designer. In this example, the instability frequencies (offset) given by the simulator are 0.887 and 2.13 GHz, which, in turn, correspond to the physical oscillation frequencies previously determined (that is, $0.881 \text{ GHz} \cong \text{DC} + 0.887 \text{ GHz}$, $0.881 \text{ GHz} \cong 3 - 2.13 \text{ GHz}$).

CONCLUSION

Traditionally used stability parameters (K and Δ) most often are not sufficient to determine the stability or instability of a two-port active microwave network, especially for nonlinear circuits, and even linear circuits with a feedback loop. Requiring all active devices to be individually unconditionally stable is most often too stringent a demand and generally does not permit the designer to achieve optimum performance from a design. Stability analysis using Nyquist and bifurcation criteria avoids such limitations and enables designers to achieve better performance from their circuit with an added level of confidence that their circuits will be stable as an amplifier or an oscillator.

ACKNOWLEDGMENT

The simulations in this article were performed using the GoldenGate suite of simulators, products of Xpedion Design Systems Inc., Santa Clara, CA. Additional information may be obtained from the company's Web site at www.xpedion.com. ■

References

1. J.M. Rollet, "Stability and Power-gain Invariants of Linear Two-ports," *IRE Transactions Circuit Theory*, Vol. 9, 1962, pp. 29-32.
2. M. Odyniec, "Oscillator Stability Analysis," *Microwave Journal*, Vol. 42, No. 6, June 1999, pp. 66-76.
3. A.P.S. Khanna, *Microwave Solid-state Circuit Design*, Chapter 9, John Wiley, 1988.
4. M.S. Nakhla and J. Vlach, "A Piecewise Harmonic Balance Technique for Determination of the Periodic Response of Nonlinear Systems," *IEEE Transactions on Circuits and Systems*, Vol. 23, February 1976, pp. 85-91.
5. E. Ngoya and R. Larcheveque, "Envelope Transient Analysis: A New Method for the Transient and Steady-state Analysis of Microwave Communication Circuits and Systems," *IEEE MTT Symposium Digest*, 1996, pp. 1365-1368.
6. *GoldenGate™ User's Manual*, Xpedion Design Systems, Santa Clara, CA.
7. Ravender Goyal (editor), *Monolithic Microwave Integrated Circuit: Technology and Design*, Chapter 5, Artech House, 1990.
8. P.N. Brown, A.C. Hindmarsh and L.R. Petzold, "Using Krylov Subspace Methods in the Solution of Large-scale Differential-algebraic Systems," *SIAM Journal on Scientific and Statistical Computing*, Vol. 15, November 1994, pp. 1467-1488.
9. D. Hente and R.H. Jansen, "Frequency Domain Continuation Method for the Analysis and Stability Investigation of Nonlinear Microwave Circuits," *IEE Proceedings*, Part H, Vol. 133, No. 5, October 1986, pp. 351-362.
10. K.S. Kundert and A. Sangiovanni-Vincentelli, "Simulation of Nonlinear Circuits in the Frequency Domain," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 5, October 1986, pp. 521-535.
11. P. Feldmann and J. Roychowdhury, "Computation of Waveform Envelopes Using an Efficient, Matrix-decomposed Harmonic Balance Algorithm," *Proceedings of the IEEE/ACM International Conference on Computer-aided Design*, November 1996, pp. 295-300.



Xpedion Design Systems, Inc.
4677 Old Ironsides Drive
Santa Clara, CA 95054
Tel: 1-877-xpedion
e-mail: info@xpedion.com
www.xpedion.com

For more information about Agilent EEs of EDA, visit:

www.agilent.com/find/eesof

 **Agilent Email Updates**

www.agilent.com/find/emailupdates
Get the latest information on the products and applications you select.

 **Agilent Direct**

www.agilent.com/find/agilentdirect
Quickly choose and use your test equipment solutions with confidence.

www.agilent.com

For more information on Agilent Technologies' products, applications or services, please contact your local Agilent office. The complete list is available at:

www.agilent.com/find/contactus

Americas

Canada	(877) 894-4414
Latin America	305 269 7500
United States	(800) 829-4444

Asia Pacific

Australia	1 800 629 485
China	800 810 0189
Hong Kong	800 938 693
India	1 800 112 929
Japan	0120 (421) 345
Korea	080 769 0800
Malaysia	1 800 888 848
Singapore	1 800 375 8100
Taiwan	0800 047 866
Thailand	1 800 226 008

Europe & Middle East

Austria	0820 87 44 11
Belgium	32 (0) 2 404 93 40
Denmark	45 70 13 15 15
Finland	358 (0) 10 855 2100
France	0825 010 700*
	*0.125 €/minute
Germany	01805 24 6333**
	**0.14 €/minute
Ireland	1890 924 204
Israel	972-3-9288-504/544
Italy	39 02 92 60 8484
Netherlands	31 (0) 20 547 2111
Spain	34 (91) 631 3300
Sweden	0200-88 22 55
Switzerland	0800 80 53 53
United Kingdom	44 (0) 118 9276201

Other European Countries:
www.agilent.com/find/contactus

Revised: March 27, 2008

Product specifications and descriptions in this document subject to change without notice.

© Agilent Technologies, Inc. 2008

Printed in USA, May 01, 2000
5989-9448EN