Measurement Uncertainty of VNA Based TDR/TDT Measurement

Application Note

Overview

Recently, vector network analyzer (VNA) based time domain reflection/time domain transmission (TDR/TDT) measurements have become very popular because of their higher accuracy and ESD robustness. Accordingly, the requirement for understanding the measurement uncertainty associated with VNA-based TDR/TDT measurements has become more critical. Although S-parameter measurement uncertainty has been studied for many years, VNA-based TDR/TDT measurement uncertainty has not been characterized as thoroughly. It is the intent of this paper to propose a robust method to calculate measurement uncertainty of VNA based TDR/TDT measurement on the basis of not only systematic instrumentation error, but also based on both time and frequency domain dependent information. As a design case study for a real world example, Agilent ENA series network analyzer with option TDR will be discussed in detail.
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Introduction

VNA based TDR/TDT measurement is becoming popular in accordance with the evolution of higher data rates of serial communications. TDR/TDT measurement is performed by using a VNA for measuring S-parameters and converting this information into the time domain by Fourier transformation. S-parameter measurement uncertainty [1] has been studied for a long time and therefore simple models and equations to calculate uncertainty have already been established. On the other hand, TDR/TDT measurement uncertainty has not been characterized as thoroughly [2].

At first, well established models and equations of S-parameter measurement uncertainty based on systematic error terms (directivity, source match, reflection tracking, transmission tracking and load match) of VNA must be explained. The systematic error terms of VNA’s are usually provided in the frequency domain. To calculate TDR/TDT measurement uncertainty, the systematic error terms of the VNA should be converted into the time domain, but it is not possible without phase information. To overcome this lack of information, this paper proposes a method to categorize frequency response into four types. The maximum value of each frequency response is converted into the maximum value in the time domain and the sensitivity coefficients from the frequency domain to the time domain is obtained. Another obstacle to calculate TDR/TDT measurement uncertainty is convolution. Multiplication in the frequency domain is convolution in the time domain and it is a complicated operation. This paper proposes a method to avoid convolution to simplify calculation. Then the measurement uncertainty can be calculated simply with the sensitivity coefficients from the frequency domain to the time domain. Finally, actual measurement uncertainties with Agilent ENA Option TDR are calculated based on the proposed method as an example.
S-parameter Measurement Uncertainty of VNA

The measurement uncertainty of S-parameters which are measured with a VNA is defined as Equation (1) [1].

\[
(\text{Measurement uncertainty}) = \sqrt{(\text{Uncertainty by Systematic effects})^2 + (\text{Uncertainty by Random effects})^2 + (\text{Uncertainty by Drift & Stability effects})^2}
\] (1)

The systematic effects are due to a systematic bias that can be quantified and corrected using vector error correction. The random effects are not repeatable such as noise and connector repeatability. The drift and stability effects are due to variations in the systematic effects over time and/or temperature.

For the TDR and TDT measurement, the random effects, drift and stability effects are much smaller than the systematic effects and are negligible. In this paper, the measurement uncertainty is defined with only the uncertainty by systematic effects. Among the systematic effects, the crosstalk term can be neglected because the contribution of it is very small (less than 10^{-5}). The systematic effects of reflection (\(S_{11}\)) and transmission (\(S_{21}\)) measurements are defined as Equation (2) and (3) [1]. The uncertainty by the systematic effects is derived by root square sum of uncertainty of each term in Equation (2) or (3).

\[
\Delta S_{11} = \delta + \tau_1 S_{11} + \mu_1 S_{11}^2 + \mu_2 S_{21} S_{12} + A S_{11}
\] (2)

\[
\Delta S_{21} = (\tau_2 + \mu_1 S_{11} + \mu_2 S_{22} + \mu_1 \mu_2 S_{21} S_{12} + A) S_{21}
\] (3)

\(S_{11}, S_{21}, S_{12}, S_{22}\) : s-parameters of device under test (DUT).

\(\delta\): Directivity
This error is due to imperfection of a coupler or a bridge of VNA and reflection from fixtures (including cables, connectors and adapters) to connect to the DUT.

\(\mu_1\): Source match
This error is due to the impedance mismatch of VNA and fixture at the source port.

\(\tau_1\): Reflection tracking
This error is due to frequency response of a coupler or a bridge, receivers (including mixers etc.) of VNA and fixtures for reflection measurement.

\(\mu_2\): Load match
This error is due to impedance mismatches of VNA and fixtures at the receiver port.

\(\tau_2\): Transmission tracking
This error is due to frequency response of a coupler or a bridge, receivers (including mixers etc.) of VNA and fixture for transmission measurement.

\(A\): Dynamic accuracy
This error is due to non-linearity (compression) of receivers of VNA.

The terms \(\delta, \mu, \tau\) and \(A\) are referred to as “error terms”, in particular they are called “residual error terms” after vector error correction is performed. The residual error terms are predominantly a function of the calibration kit. Normally specifications of all the residual error terms are provided in the frequency domain.
Simplification of Systematic Effects Equations

Figure 1 shows a signal flow graph of a TDR measurement including error models which consist of error terms. The response time of the term $\mu_2 S_{21} S_{12}$ of TDR measurement ($S_{11}$) is round trip time of the DUT ($2 t_{DUT}$). $t_{DUT}$ is the earliest time when step stimulus reaches $\mu_2$. For a TDR measurement, the response from outside of the DUT is not the focus of interest. So the term $\mu_2 S_{21} S_{12}$ can be neglected.

Figure 1. Signal flow graph of TDR measurement

\[ \Delta S_{11} = \delta + \tau_1 S_{11} + \mu_1 S_{11}^2 + \mu_2 S_{21} S_{12} + A S_{11} \]

Figure 2 shows a signal flow graph of a TDT measurement. The response time of the term $\mu_1 \mu_2 S_{21} S_{12} S_{21}$ of the TDT measurement ($S_{21}$) is three times of $t_{DUT}$. For a TDT measurement, it is not the focus of interest and the term $\mu_1 \mu_2 S_{21} S_{12} S_{21}$ can be neglected.

Figure 2 shows a signal flow graph of a TDT measurement.
By neglecting the terms which are not the focus of interest, the equations of the systematic effects are simplified (see below) in the frequency domain.

\[ \Delta S_{21} = (\tau_2 + \mu_1 S_{11} + \mu_2 S_{22} + \mu_1 \mu_2 S_{21} S_{12} + A) S_{21} \]

\[ \Delta S_{11} = \delta + \tau_1 S_{11} + \mu_1 S_{11}^2 + A S_{11} \quad (4) \]

\[ \Delta S_{22} = (\tau_2 + \mu_1 S_{11} + \mu_2 S_{22} + A) S_{22} \quad (5) \]
Conversion from Frequency Domain into Time Domain

The specifications of the residual error terms of VNA and Calibration kit are usually provided in the frequency domain. Table 1 is an example of specifications of a calibration kit.

Table 1. Specifications of Agilent N4433A Electronic Calibration (ECal) Module

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>300 kHz to 10 MHz</th>
<th>10 MHz to 5 GHz</th>
<th>5 GHz to 9 GHz</th>
<th>9 GHz to 13.5 GHz</th>
<th>13.5 GHz to 20 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directivity (dB)</td>
<td>45</td>
<td>52</td>
<td>47</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>Source match (dB)</td>
<td>36</td>
<td>42</td>
<td>39</td>
<td>37</td>
<td>31</td>
</tr>
<tr>
<td>Reflection tracking (± dB)</td>
<td>0.1</td>
<td>0.06</td>
<td>0.09</td>
<td>0.1</td>
<td>0.18</td>
</tr>
<tr>
<td>Transmission tracking (± dB)</td>
<td>0.078</td>
<td>0.045</td>
<td>0.057</td>
<td>0.069</td>
<td>0.16</td>
</tr>
<tr>
<td>Load match (dB)</td>
<td>39</td>
<td>45</td>
<td>41</td>
<td>39</td>
<td>35</td>
</tr>
</tbody>
</table>

For most network analyzers, the specifications of the systematic effects are guaranteed values and they should be the maximum values in the frequency domain. In this paper, the maximum values in the frequency domain will be converted into the maximum values in the time domain. Standard deviations of systematic effects are required to calculate uncertainty, but distribution of the systematic effect of the network analyzer is usually unknown, and uniform distribution is used to be on the safe side. For the uniform distribution, the standard deviation is calculated as the maximum value divided by $\sqrt{3}$. When the distribution of values in the time domain is assumed to be uniform and $1/\sqrt{3}$ is used to calculate standard deviation, sensitivity coefficient of uncertainty from the frequency domain to the time domain is the same as the conversion coefficient of the maximum value from the frequency domain to the time domain. In the case of other distribution, the coefficients need to be adjusted, but the uncertainty in the time domain is independent from the distribution in the frequency domain, because calculation of the uncertainty in the time domain is based on the maximum value in the frequency domain.

The purpose of this chapter is to calculate the maximum value of the time domain step response $|x(t)|_{\text{max}}$ from the maximum value of frequency response $|X(f)|_{\text{max}}$ which is provided in Table 1. $X(f)$ and $x(t)$ represent one of the error terms in Equation (4) or (5) in the frequency domain and the time domain respectively. For our purposes here, the time domain response means the time domain step response when it is not clearly stated. The time domain step response is a time integral of inverse discrete Fourier transformation of frequency response. It is defined in following.

$$x(t) = \sum_{n=-N}^{N} X(f_n) e^{j 2\pi n f_1 t} \left( -j \cos (2\pi n f_1 t) + \sin (2\pi n f_1 t) \right)$$

(6)

$x(t)$: time domain step response
$X(f)$: frequency response
$X(0)$: DC response (at $f = 0$)
$f_1$: the lowest measurement frequency (= frequency step of frequency response)
$N$: number of measurement points of $X(f)$
The maximum value does not have information of actual frequency response \(X(f_n)\) at each frequency \(f_n\), and discrete Fourier transformation it is not possible without actual frequency response, but time domain step responses can be calculated for specific types of frequency response. The maximum value in the time domain occurs in four types of frequency response or their combinations. (a.) DC response which is converted into a ramp function in the time domain with the maximum value at the maximum time. (b.) Constant frequency response which is converted into a step function with a constant value at \(t > 0\). (c.) Integral type frequency response which is converted into a ramp function. (d.) Derivative type frequency response which is converted into an impulse function. In the following sections, the four types of frequency response are discussed in detail.

a. DC response

Because the DC response of the DUT can’t be measured directly with a VNA, the DC response value is estimated with the measurement results of other frequencies. Alternately, the user may be able to input DC value directly. In this paper, it is assumed that the error terms of the DC value is less than or equal to the error terms in the low frequency area.

\[
|X(0)| \leq |X(f)|_{\text{max at LF}} \tag{7}
\]

The time domain step response can be converted from DC value \(X(0)\).

\[
x(t) = X(0) f_1 t \tag{8}
\]
Equation (8) shows that $x(t)$ is a ramp function that becomes larger and larger with an increase in time, but the maximum value is limited by observation time. For the VNA based TDR/TDT measurement, the maximum time span of observation is $1/f_1$ ($f_1$ is the lowest measurement frequency). Normally the full range of the time span is not the focus of interest. When the focus of interest is limited from $t = 0$ to $1/m$ of maximum time span ($1/f_1$), the maximum value of $|x(t)|$ is limited by $(1/m)$ as following.

$$|x(t)| \leq \frac{1}{m} |x(0)| \leq \frac{1}{m} |x(f)|_{\text{max of LF}} \quad \text{if } t \leq \frac{1}{mf_1}$$

(9)

The maximum value of $|x(t)|$ by DC component can be expressed as follows.

$$|x(f)|_{\text{max of DC}} = \frac{1}{m} |x(f)|_{\text{max of LF}}$$

(10)

b. Constant frequency response

When frequency response $X(f)$ is constant, the time domain step response $x(t)$ is a step function. The height of $x(t)$ is same as $X(f)$. If $X(f) = 1.0$ (constant), $x(t) = 1.0 \ (t > 0)$.

![Figure 4. Conversion of constant frequency response](image)
If there is phase variation in the frequency response, the shape of the time domain step response deviates from the step function. Extensive simulations with various non-linear phase response show that the maximum value in the time domain varies up to 3/2 times of $X(f)$ as shown in Figure 5. The left graph is the constant frequency response and the right graph is time domain response with various phase response. This simulation was performed with the data points of 1,000 and the phase was assumed to vary continuously across the frequency.

For the time domain step response, lower frequency response contributes more than higher frequency response. So the maximum value by constant frequency response is derived as following using the maximum value of lower frequency.

\[ |x(f)| \max \text{ of Const} = \frac{3}{2} |x(f)| \max \text{ of LF} \]  

(11)
c. Integral type frequency response

In the case of the integral type frequency response, the time domain step response is a ramp function. \(|X(f)|\) is inverse proportional to frequency and can be expressed as Equation (12) and converted into the time domain step response as Equation (13).

\[
|X(f)| = \frac{\omega_0}{2\pi f}
\]

(12)

\[
|x(t)| = \omega_0 t
\]

(13)

\(\omega_0\) can be derived from Equation (12) by using frequency response at frequency \(f_i\). Substituting the \(\omega_0\) into Equation (13), \(|x(t)|\) can be expressed with \(|X(f_i)|\). \(|X(f_i)|\) is the maximum value of the frequency response.

\[
\omega_0 = 2\pi f_i \ |X(f_i)|
\]

\[
|x(t)| = 2\pi f_i \ t \ |X(f_i)| = 2\pi f_i \ t \ |X(f)| \text{ max at LF}
\]

(14)

The maximum value is limited by observation time as mentioned in DC response type. When the focus of interest is limited from \(t = 0\) to \(1/m\) of maximum time span \((1/f_i)\), the maximum value of \(|x(t)|\) is limited by \((1/m)\). So the maximum value by the integral type frequency response is determined by Equation (15).

\[
|X(t)| \text{ max by Integ} = \frac{2\pi}{m} \ |X(f)| \text{ max at LF}
\]

(15)
The equation (15) is valid even if there is phase variation in the frequency response. Extensive simulations with various phase response are shown in Figure 7. The left graph is the integral type frequency response and the right graph is time domain response with various phase response. The maximum value is the same as derived in Equation (15).

Figure 7. Conversion of integral type frequency response
d. Derivative type frequency response

In the case of derivative type frequency response, the time domain step response is the impulse function. If \( x(t) \) is maximum at \( t = 0 \), the value \( x(0) \) is expressed as Equation (16).

\[
x(0) = \frac{1}{\pi} \sum_{n=1}^{N} \frac{X(f) \cos(0)}{n} = \frac{1}{\pi} \sum_{n=1}^{N} \frac{X(f)}{n}
\]  

(16)

Since \( X(f) \) is proportional to \( n \), \( X(f)/n \) is constant and it is equal to \( X(f_0) \) regardless of \( n \) and \( N \). Equation (16) is simplified as follows.

\[
x(0) = \frac{1}{\pi} \sum_{n=1}^{N} X(f_n) = \frac{1}{\pi} N \cdot X(f_0) = \frac{1}{\pi} X(f_0)
\]  

(17)
Because of the truncation at the upper limit of the frequency response, the actual time domain response is not a pure impulse (width = 0) but it has a finite width. Based on extensive simulations with various pulse width, the maximum value varies up to 2/3 of $|X(f)|_{\text{max}}$. Figure 9 shows an example of the simulation results. The left graph is the frequency response (in log scale) and the right graph is time domain response with various pulse widths. For this type of response, the maximum value of the time domain is determined by the higher frequency response and it can be expressed as Equation (18).

\[
|X(t)|_{\text{max by Deriv}} = \frac{2}{3} |X(f)|_{\text{max at HF}}
\]  

\((18)\)
The maximum value by ripple type frequency response can be expressed with the above equation, because the ripple type frequency response can be included in the derivative type frequency response. A ripple type frequency response occurs when there is more than one discontinuity on the DUT. The frequency of the ripple depends on the distance of two discontinuities. The distance of two discontinuities means pulse width in the time domain. When deriving Equation (18), not only the pure impulse (width = 0) but also various pulse width responses are considered. So the equation of the maximum value by the ripple type frequency response is the same as Equation (18) for the derivative type frequency response. The extensive simulations in Figure 10 verify this fact. The graph on the left is the frequency response (in linear scale) and the graph on the right is the time domain response with various ripple shapes.

Figure 10. Conversion of ripple type frequency response
e. Maximum value of time domain

In the previous sections, four maximum values in the time domain are derived from the five types of frequency response.

\[ |x(t)|_{\text{max by DC}} = \frac{1}{m} |X(f)|_{\text{max at LF}} \]
\[ |x(t)|_{\text{max by Const}} = \frac{3}{2} |X(f)|_{\text{max at LF}} \]
\[ |x(t)|_{\text{max by Integ}} = \frac{2\pi}{m} |X(f)|_{\text{max at LF}} \]
\[ |x(t)|_{\text{max by Deriv}} = \frac{2}{3} |X(f)|_{\text{max at HF}} \]

(19)

\( m \) is a ratio of observation time of interest to maximum observation time \((1/f_1)\).

Since the integral type frequency response generates a ramp waveform and it includes a ramp waveform by the DC response, \(|x(t)|_{\text{max by DC}}\) does not need to be added for the time domain maximum value. \(|x(t)|_{\text{max by Const}}\) and \(|x(t)|_{\text{max by Integ}}\) are determined by lower frequency response and the conditions for the both values can’t be satisfied simultaneously. So only the larger value can be adopted. Normally, \( m \) is about 1/4 for TDR/TDT measurement and \(|x(t)|_{\text{max by Integ}}\) is larger than \(|x(t)|_{\text{max by Const}}\). \(|x(t)|_{\text{max by Deriv}}\) is independent from other values. For the most of VNAs and calibration kits, the specifications of middle frequency are better than or equal to the specification of lower frequency \(|X(f)|_{\text{max at LF}}\) and/or higher frequency \(|X(f)|_{\text{max at HF}}\). So contribution of middle frequency can be included in one of the maximum values in Equation (19). Finally, the maximum value in the time domain can be determined by combination of the two maximum values in the frequency domain as following-

\[ |x(t)|_{\text{max}} = \left( \frac{\pi}{2} \right) |X(f)|_{\text{max at LF}} + \left( \frac{2}{3} \right) |X(f)|_{\text{max at HF}} \]

(20)

To calculate the uncertainty of \(x(t)\), the uncertainties of the two terms in the above equation are combined by root square sum because the effect of LF and HF are independent from each other. The sensitivity coefficient from \(X(f)_{\text{at LF}}\) to \(x(t)\) is \((\pi/2)\) and the sensitivity coefficient from \(X(f)_{\text{at HF}}\) to \(x(t)\) is \((2/3)\).
f. Conversion example with actual data

In this section, the method described above (a. to f.) is verified with the real world data. The real world frequency response data is converted into the time domain and it is compared with the calculated maximum value which is converted from the maximum value in the frequency domain. As the real world data, one of error coefficients is used instead of error terms. The actual error terms can’t be obtained because the true values of the calibration kit’s response are required but they are unknown. Error coefficients of a VNA are quantified values during calibration process and they behave similarly as residual error terms.

The blue trace in Figure 11 is the actual error coefficient (directivity) in the frequency domain of a VNA. Figure 12 is the time domain value which is converted from Figure 11. \( |X(f)|_{\text{max at LF}} \) (red and solid line) is the maximum value at lower frequency and it determines \( |x(t)|_{\text{max by Integ.}} \). \( |X(f)|_{\text{max at HF}} \) (purple and dashed line) is the maximum value at higher frequency and it determines \( |x(t)|_{\text{max by Deriv.}} \). The maximum value in the time domain \( |x(t)|_{\text{max}} \) is the sum of the two maximum values. Figure 12 shows that the actual time domain response is considerably smaller than \( |x(t)|_{\text{max}} \). This means derived value \( |x(t)|_{\text{max}} \) can be used as the maximum value in the time domain.

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**Figure 11. Error coefficient in the frequency domain**

**Figure 12. Error coefficient in the time domain**
Simplification by eliminating Convolution

In the previous chapter, the maximum value in the frequency domain can be converted into the maximum value in the time domain, but there is another obstacle of convolution. Addition in the frequency domain is also addition in the time domain, but multiplication in the frequency domain is convolution in the time domain. For example, directivity $\delta$ in Equation (21) is apparently offset term in both the frequency domain and the time domain. On the other hand, $\tau_1$ in Equation (21) is a proportional term for $S_{11}$ in the frequency domain, but it is a convolution term in the time domain. Convolution is a complex operation and it is not suitable for a simple expression.

\[ \Delta S_{11} = \delta + \tau_1 \ast S_{11} + \mu_1 \ast S_{11}^2 + A \ast S_{11} \]  \hspace{1cm} (21)

\(^\ast\) denotes multiplication in the frequency domain and convolution in the time domain.

All the systematic errors of $\Delta S_{21}$ (noted as $\epsilon$) are proportional to $S_{21}$ in the frequency domain.

\[ \Delta S_{21} = (\tau_2 + \mu_1 \ast S_{11} + \mu_3 \ast S_{22} + A) \ast S_{21} = \epsilon \ast S_{21} \]
\[ \epsilon = \tau_2 + \mu_1 S_{11} + \mu_3 S_{22} + A \]  \hspace{1cm} (22)

Generally $\epsilon$ is a convolution term in the time domain, but if the time domain step response of $S_{21}$ is a monotonically increasing function and the maximum value of $\epsilon$ is known as $|\epsilon|_{max}$, $|\epsilon|_{max}$ can be a proportional term for $|\Delta S_{21}(t)|_{max}$ in the time domain.

\[ |\Delta S_{21}(t)|_{max} = |\epsilon(t)|_{max} S_{21}(t) \]  \hspace{1cm} (23)

It is derived as follows:

The step response of $\Delta S_{21}$ is equal to convolution of the time domain step response of $\epsilon$ and impulse response of $S_{21}$.

\[ \Delta S_{21}^{step}(nT_s) = \epsilon^{step} \otimes S_{21}^{impulse} \]
\[ = \sum_{k=-\infty}^{\infty} \epsilon^{step} ((n-k)T_s) S_{21}^{impulse} (nT_s) \]  \hspace{1cm} (24)

$T_s$ : time interval of the time domain step response
$S_{21}^{step}$ : time domain step response of $S_{21}$
$S_{21}^{impulse}$ : time domain impulse response of $S_{21}$
$\epsilon^{step}$ : time domain step response of $\epsilon$
$\otimes$ : denotes convolution
Because $\epsilon_{\text{step}}$ should be causal, it is 0 at $t < 0$ and the range of $k$ can be limited between $-\infty$ to $n$. $|\epsilon_{\text{step}}|_{\text{max}}$ is the maximum value of $\epsilon_{\text{step}}$. $|\epsilon_{\text{step}}|_{\text{max}}$ can be calculated by the method explained in the previous chapter. $\Delta S_{21,\text{step}}$ is expressed as follows.

$$
\Delta S_{21,\text{step}}(nT_s) = \sum_{k=-\infty}^{\infty} \epsilon_{\text{step}}((n - k)T_s) S_{21,\text{impulse}}(nT_s)
$$

$$
\leq \sum_{k=-\infty}^{\infty} |\epsilon_{\text{step}}((n - k)T_s)| |S_{21,\text{impulse}}(nT_s)|
$$

$$
\leq |\epsilon_{\text{step}}|_{\text{max}} \sum_{k=-\infty}^{\infty} |S_{21,\text{impulse}}(nT_s)|
$$

(25)

When $S_{21,\text{impulse}}$ is always 0 or positive, $|S_{21,\text{impulse}}|$ is same as $S_{21,\text{impulse}}$.

$$
\Delta S_{21,\text{step}}(nT_s) \leq |\epsilon_{\text{step}}|_{\text{max}} \sum_{k=-\infty}^{\infty} S_{21,\text{impulse}}(nT_s) = |\epsilon_{\text{step}}|_{\text{max}} S_{21}(nT_s)
$$

$$
\Delta S_{21,\text{step}}|_{\text{max}} = |\epsilon_{\text{step}}|_{\text{max}} S_{21,\text{step}}(t) \quad \text{where} \quad t = nT_s
$$

(26)

This means the time domain maximum value $|\epsilon_{\text{step}}|_{\text{max}}$ can be a proportional coefficient of the time domain step response of $S_{21}$. 0 or positive value of impulse response means that step response of $S_{21}$ should be a monotonically increasing function. Actual time domain step response of $S_{21}$ may have a small dip caused by multiple reflections, but it is almost a monotonically increasing function from a macroscopic viewpoint. So, $|\epsilon_{\text{step}}|_{\text{max}}$ behaves as a proportional error for the time domain response of $S_{21}$.

For the case of $S_{11}$, the same method can’t be applied because the step response of $S_{11}$ is not a monotonic function. When the maximum value of $S_{11}$ in the frequency domain is known (like $|S_{11}| < -20\text{dB}$), the equation can be simplified. For example, the maximum value of $\tau S_{11}$ can be calculated in the frequency domain. After the multiplication, the value can be converted into the time domain by the method in the previous chapter. By eliminating convolution, $\Delta S_{11}$ depends on only the maximum value of $S_{11}$ and it is independent from each value of $S_{11}$. This means $\Delta S_{11}$ behaves as an offset error for $S_{11}$. 
Figure 13 shows steps to convert the systematic effects in the frequency domain into the time domain. The uncertainties are calculated by root square sum of uncertainty of each term in Figure 13.

\[ \Delta S_{11} = \delta + \tau_1 S_{11} + \mu_1 S_{11}^2 + A S_{11} \]

1 : Determine the maximum value of \( S_{11} \) and \( S_{22} \). Then multiply the maximum value of \( S_{11} \) or \( S_{22} \) with \( \tau_1 \), \( \mu_1 \), \( \mu_2 \), and \( A \).

2 : Convert each term into the time domain by the method described in the previous chapter and add all terms. For \( \Delta S_{11} \), the sum of all terms is offset error.

3 : For \( \Delta S_{21} \), the sum of all terms is proportional error.

\[ \Delta S_{21} = (\tau_2 + \mu_1 S_{11} + \mu_2 S_{22} + A) S_{21} \]

Figure 13. Summary of conversion steps
Example of Measurement Uncertainty Calculation

In this section, the real uncertainties of TDR and TDT measurement are calculated from the specifications of the Agilent E5071C ENA [3] Option TDR and Agilent N4433A ECal module [4] as an example.

The specifications of the residual error terms of N4433A ECal module are shown in Table 1. In Table 2, the uncertainties converted from the specifications are shown. To convert the specifications into the uncertainties, the specifications of directivity, source match and load match in Table 1 are converted into linear numbers and inversed \((1/x)\) and divided by \(\sqrt{3}\) (uniform distribution is assumed). The reflection tracking and transmission tracking are converted into linear numbers and 1.0 is subtracted from them and divided by \(\sqrt{3}\). Only the two frequency ranges of interest are shown in Table 2. The uncertainties of middle frequency (5 GHz to 13.5 GHz) are better than the uncertainties of the higher frequency (13.5 GHz to 20 GHz).

Table 2. Uncertainties of residual error terms of Agilent N4433A ECal Module

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>10 MHz to 5 GHz</th>
<th>13.5 GHz to 20 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directivity (dB)</td>
<td>(\delta)</td>
<td>0.0015</td>
</tr>
<tr>
<td>Source match (dB)</td>
<td>(\mu_1)</td>
<td>0.0046</td>
</tr>
<tr>
<td>Reflection tracking (± dB)</td>
<td>(\tau_1)</td>
<td>0.0040</td>
</tr>
<tr>
<td>Transmission tracking (± dB)</td>
<td>(\tau_2)</td>
<td>0.0030</td>
</tr>
<tr>
<td>Load match (dB)</td>
<td>(\mu_2)</td>
<td>0.0032</td>
</tr>
</tbody>
</table>

Table 3 and Table 4 show calculated uncertainties of TDR and TDT measurement with E5071C which is calibrated using the N4433A. The conditions are as follows.

\[ S_{11}, S_{21} < 0.1 \]
\[ -20 \text{ dB; 40.91 } \Omega \text{ to 61.11 } \Omega \]
\[ f_1 = 10 \text{ MHz, } N = 2000 \]
\[ (1/f_1 = 100 \text{ ns, } m = 1/4) \]
\[ \text{Dynamic Accuracy of ENA}[3]; 0.045 \text{ dB} \]
\[ \text{(at } -30 \text{ dBm @ } -10 \text{ dBm reference)} \]

The uncertainty of the dynamic accuracy of ENA is 0.0030 which is obtained by converting 0.045 dB into a linear number and 1.0 is subtracted from it and divided by \(\sqrt{3}\).

In Table 3 and Table 4, all the frequency domain uncertainties in Table 2 are converted into the time domain uncertainties by multiplying with the sensitivity coefficients \((\pi/2 \text{ for LF and } 2/3 \text{ for HF})\) described in the section 4-e.
Table 3. TDR measurement uncertainty

<table>
<thead>
<tr>
<th>TDR measurement</th>
<th>Time domain uncertainty</th>
<th>Converted from LF</th>
<th>Converted from HF</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directivity</td>
<td>δ</td>
<td>0.0023</td>
<td>0.0022</td>
<td>0.0031</td>
</tr>
<tr>
<td>Source match</td>
<td>μ₁ S₁¹</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>Reflection tracking</td>
<td>τ₁ S₁¹</td>
<td>0.0006</td>
<td>0.0008</td>
<td>0.0010</td>
</tr>
<tr>
<td>Dynamic accuracy</td>
<td>A S₁¹</td>
<td>0.0005</td>
<td>0.0002</td>
<td>0.0005</td>
</tr>
<tr>
<td>Combined standard uncertainty (k = 1)</td>
<td></td>
<td></td>
<td></td>
<td>0.0033</td>
</tr>
<tr>
<td>Expanded uncertainty (k = 2)</td>
<td></td>
<td></td>
<td></td>
<td>0.0067</td>
</tr>
</tbody>
</table>

TDR measurement uncertainty is ± 0.0067 as offset for TDR measurement. This means the measurement uncertainty of 50 Ω impedance is ± 0.67 Ω. The directivity term is dominant for the TDR measurement uncertainty and it can be improved with more accurate calibration like TRL calibration.

Table 4. TDT measurement uncertainty

<table>
<thead>
<tr>
<th>TDT measurement</th>
<th>Time domain uncertainty</th>
<th>Converted from LF</th>
<th>Converted from HF</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directivity</td>
<td>μ₁ S₁¹</td>
<td>0.0007</td>
<td>0.0011</td>
<td>0.0013</td>
</tr>
<tr>
<td>Source match</td>
<td>μ₂ S₂²</td>
<td>0.0005</td>
<td>0.0007</td>
<td>0.0009</td>
</tr>
<tr>
<td>Reflection tracking</td>
<td>τ₂ S₂²</td>
<td>0.0047</td>
<td>0.0072</td>
<td>0.0086</td>
</tr>
<tr>
<td>Dynamic accuracy</td>
<td>A S₂²</td>
<td>0.0047</td>
<td>0.0020</td>
<td>0.0051</td>
</tr>
<tr>
<td>Combined standard uncertainty (k = 1)</td>
<td></td>
<td></td>
<td></td>
<td>0.0101</td>
</tr>
<tr>
<td>Expanded uncertainty (k = 2)</td>
<td></td>
<td></td>
<td></td>
<td>0.0202</td>
</tr>
</tbody>
</table>

TDT measurement uncertainty is ± 2.02% of transmission parameter.

Recent oscilloscopes can be calibrated with a calibration kit and measurement uncertainty can be calculated similarly using specifications of the calibration kit, but the dynamic range of oscilloscope is usually narrower than VNA and random errors need to be counted.
Conclusions

In this application note, a method to derive measurement uncertainty of VNA-based TDR/TDT measurement is proposed. To convert the frequency domain response into the time domain step response, a method to categorize frequency response into four types is described. The sensitivity coefficients from the frequency domain to the time domain are obtained for each type of frequency response. These coefficients are used to convert the uncertainty in the frequency domain into the time domain. The uncertainties in the frequency domain are derived from the specifications of residual error terms of calibration kit and VNA. Also described is a method to eliminate convolution in the time domain to simplify the measurement uncertainty.

With the proposed method, the measurement uncertainty can be calculated simply from the specifications of VNA and calibration kit.

Finally the actual TDR/TDT measurement uncertainties with Agilent ENA Option TDR calibrated with N4433A ECal module are calculated based on the proposed method as an example.

References


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