When purchasing an oscilloscope to test new designs, the primary performance specification that most engineers consider first is the scope’s real-time bandwidth. “Real-time” simply refers to the scope’s ability to deliver the specified bandwidth on each and every acquisition cycle, or trigger event. In other words - real-time bandwidth is basically the same as single-shot bandwidth - not repetitive or equivalent-time bandwidth.

Closely related to a scope’s real-time bandwidth- and also another key consideration - is the scope’s maximum specified sample rate. Many engineers believe that higher sample rate scopes produce more accurate measurement results since more closely-spaced samples will provide increased timing resolution, and hence a more continuous waveform display. Although this is may be true for scopes that don’t have the ability to perform digital waveform reconstruction/filtering, nearly all of today’s digital storage oscilloscopes automatically perform $\sin(x)/x$ digital filtering to provide very high-density reconstructed samples based on the Nyquist sampling theorem.

Agilent Technologies’ newest 1-GHz bandwidth oscilloscopes in the InfiniiVision 3000 X-Series sample at 5 GSa/s in a half-channel interleaved mode, or 2.5 GSa/s when using all channels of the scope. This means that these scopes’ sample rate-to-bandwidth ratio can be as low as 2.5:1. This may sound like a stretch to many, and would definitely put Nyquist to the test. So let’s put the scope to a test by capturing some high frequency analog and high-speed digital signals that would stress any 1-GHz bandwidth scope’s measurement capabilities to its limits. We will then follow-up these empirical tests with the supporting theory behind it.
High Frequency Analog Applications

Figure 1 shows a 1-GHz sine wave captured single-shot on a 1-GHz bandwidth oscilloscope while sampling at just 2.5 GSa/s, but without waveform reconstruction. As you can see, the samples (enhanced for view ability) are widely spaced (400 ps apart). Although the front-end analog hardware of this scope delivered the input signal to the scope’s analog-to-digital conversion acquisition system with 1-GHz bandwidth (less than 3 dB of attenuation), 400 ps of sample resolution is clearly insufficient for creating a waveform that can be visually interpreted. To most, this probably just looks like a useless scatter of dots. Although this may be what many engineers would expect to see on a scope’s display with the timebase set at 500 ps/div while sampling at just 2.5 GSa/s, this is not what the Agilent InfiniVision 3000 X-Series scope displays.

Figure 2 shows what the 1-GHz bandwidth MSOX3104A oscilloscope displays when digitizing a 1-GHz sine wave at 2.5 GSa/s at the same timebase setting (500 ps/div). The scope’s automatic sin(x)/x waveform reconstruction filter enhances the sampled data resolution from 400 ps down to 20 ps. This provides an “effective” sample rate of 50 GSa/s. This is what some scopes on the market today call “interpolated sample rate”. And then with the scope’s automatic connect-the-dots (linear reconstruction between the 20 ps reconstructed samples), the scope displays a continuous waveform with very high resolution and display quality that you might expect from a much higher sample rate scope of the same bandwidth.

When performing a rise time measurement on this single-shot acquisition, the scope measured 300 ps. The theoretical rise time of a pure 1-GHz sine wave is approximately 295 ps based on 10% to 90% threshold criteria. Although not shown in this paper, when this 1-GHz sine wave was captured repetitively, the rise time measurement resolution was ± 20 ps. This means that for any random single-shot acquisition, the measurement could be as low as 280 ps or as high as 320 ps.

As mentioned earlier, Agilent’s 3000 X-Series oscilloscopes automatically perform sin(x)/x waveform reconstruction filtering. This occurs whenever the scope’s timebase is set to a range such that there would be less than 500 unfiltered sample points across screen. When the scope is sampling at 5 GSa/s (half-channel mode), this occurs at 5 ns/div and faster. When the scope is sampling at 2.5 GSa/s (full-channel mode), this occurs at 10 ns/div and faster.

So what are the tradeoffs of sin(x)/x waveform reconstruction? For Agilent’s 3000 X-Series oscilloscopes, the answer is none. This is why it is automatic. The user cannot disable this function of the scope. When viewing waveforms on the scope’s faster timebase ranges, sin(x)/x reconstruction always provides increased measurement resolution, increased measurement accuracy, and improved display quality; without any degradation in waveform update rate. When it comes to other scopes on the market today, the biggest tradeoff is often waveform update rate since this function of the scope is typically performed by software within the scope’s CPU system. But Agilent’s 3000 X-Series scopes perform sin(x)/x waveform reconstruction via DSP hardware, which is part of this scope’s exclusive MegaZoom IV ASIC technology. Waveform update rates can be as high as 1,000,000 waveforms/sec, which is the fastest in the oscilloscope industry, even when the scope automatically engages the sin(x)/x waveform reconstruction filter. To learn more about sin(x)/x waveform reconstruction filtering, refer to the application note listed at the end of this document titled, “Advantages and Disadvantages of Using DSP Filtering on Oscilloscope Waveforms”.

Figure 1: Single-shot capture of a 1-GHz sine wave sampled at 2.5 GSa/s, but without digital waveform reconstruction filtering.

Figure 2: Single-shot capture of a 1-GHz sine wave at 2.5 GSa/s with sin(x)/x reconstructed samples.
High-speed Digital Applications

We have just shown that a 1-GHz bandwidth scope sampling at just 2.5 GSa/s can produce a very high resolution display — far beyond the unfiltered 400 ps sample resolution — of a high frequency analog signal consisting of a single component of frequency (1 GHz sine wave). But what about high-speed digital applications, where fast transition edges can be composed of a wide spectrum of frequencies? Can sampling at 2.5 GSa/s produce an accurate and highly resolved picture of a fast transition within the bandwidth constraints of a 1-GHz bandwidth scope? Let’s see.

Figure 3 shows a single-shot capture of a fast digital transition sampled at 2.5 GSa/s, but without digital waveform reconstruction. The original 20% to 80% rise time of the input signal was in the range of approximately 350 ps. This is about the fastest edge that you could expect to obtain reasonably accurate measurements on using a 1-GHz bandwidth oscilloscope, regardless of its sample rate. To capture digital signals faster than this you should use a higher bandwidth scope.

Again, without sin(x)/x waveform reconstruction, sampling at just 2.5 GSa/s (400 ps unfiltered sample resolution) does not provide us with sufficient timing resolution to make accurate measurements on this signal. And display quality could be judged as poor-at-best since the widely-spaced samples (enhanced for view ability) don’t produce a continuous waveform, which is what most engineers would prefer to see on their oscilloscopes.

Figure 4 shows what the Agilent MSOX3104A 1-GHz bandwidth oscilloscope displays when capturing this fast rising edge while sampling at 2.5 GSa/s. The scope’s automatic sin(x)/x waveform reconstruction filter increases the sample resolution from 400 ps to 20 ps. When capturing this signal repetitively, the scope measured an average rise time of 395 ps with a worst-case measurement resolution of ± 30 ps resulting in a standard deviation of 11 ps.

So what would we see and measure if the scope’s sample rate were doubled from 2.5 GSa/s to 5 GSa/s? We can easily find out since when just half the channels of this scope are turned on, maximum sample rate can be increased to 5 GSa/s.

Figure 5 shows the results of capturing this same fast rising edge when sampling at 5 GSa/s. Although intuitively many would believe that 5 GSa/s has got to be a lot better than 2.5 GSa/s, as you can see, there’s not much difference.
The average measured rise time is just 7 ps faster, the worst-case measurement resolution is the same (± 30 ps), and the standard deviation of this measurement improved by just 2 ps. If this scope were able to sample at 10 GSa/s or even 20 GSa/s, you would see virtually no improvement at these higher sample rates for a 1-GHz bandwidth oscilloscope.

In some cases, higher sample rates can actually induce degraded measurement accuracy due to improper alignment of interleaved ADCs, which is a common design technique employed by many oscilloscope vendors, including Agilent, to produce the highest possible sample rates (up to 80 GSa/s). To learn more about interleaved sampling error, refer to the application note listed at the end of this document titled, “Evaluating Oscilloscope Sample Rates vs. Sampling Fidelity”.

Now that we have seen measurement examples that stress the measurement limits of a 1-GHz bandwidth scope, let’s now address the theory behind why 2.5 GSa/s is sufficient for a 1-GHz bandwidth scope.

**Understanding Nyquist’s sampling theorem and how it relates to oscilloscopes**

Dr. Harry Nyquist (Figure 6) postulated:

**Nyquist Sampling Theorem**
For a limited bandwidth signal with a maximum frequency \( f_{MAX} \), the equally-spaced sampling frequency \( f_s \) must be greater than twice the maximum frequency \( f_{MAX} \), in order to have the signal be uniquely reconstructed without aliasing.

Nyquist’s sampling theorem can be summarized into two simple rules:

1. The highest frequency component sampled must be less than half the sampling frequency (sample rate).
2. Samples must be equally spaced.

If these rules are adhered to, then a scope can accurately reconstruct a digitally-sampled waveform using \( \sin(x)/x \) digital filtering based on widely-spaced samples.

What Nyquist calls \( f_{MAX} \) is what we usually refer to as the Nyquist frequency \( (f_N) \), which is not the same as oscilloscope bandwidth \( (f_{BW}) \). If an oscilloscope’s bandwidth is specified exactly at the Nyquist frequency \( (f_N) \), this implies that the oscilloscope has an ideal brick-wall frequency response that falls off exactly at this same frequency as shown in Figure 7. Frequency components below the Nyquist frequency are perfectly passed through from the scope’s front-end analog hardware to the scope’s ADC acquisition system with no attenuation, and frequency components above the Nyquist frequency are perfectly eliminated. Unfortunately, this type of frequency response filter is impossible to implement in oscilloscope hardware.

**Figure 7: Theoretical brick-wall frequency response.**

**Figure 6: Dr. Harry Nyquist, 1889-1976, articulated his sampling theorem in 1928.**
Most oscilloscopes with bandwidth specifications below 1 GHz have what is known as a Gaussian frequency response as represented by the red curve shown in Figure 8. This type of response approximates the characteristics of a single-pole low-pass filter. Many higher bandwidth scopes exhibit what is called a high-order maximally-flat frequency response as represented by the green curve shown in Figure 8. This type of response begins to approach the characteristics of a theoretical brick-wall filter. In either case (Gaussian or maximally-flat), signals can be attenuated by as much as 3 dB (~30%) at the specified bandwidth frequency (f_{BW}).

If a scope’s bandwidth (f_{BW}) is specified exactly at the Nyquist frequency (f_N), which would imply a sample rate that is exactly twice the bandwidth, input signal frequency components above this frequency can be sampled, thereby resulting in aliasing (inaccurate waveform reconstruction). For a 1-GHz bandwidth scope that exhibits a maximally-flat frequency response (green curve), minimizing the possibility of sampling frequency components above the Nyquist frequency requires that the Nyquist frequency (f_{N1}) be no lower than 1.25 GHz with a minimum sample rate of 2.5 GSa/s. This means that the minimum sample rate-to-bandwidth ratio be no lower than 2.5:1. Agilent’s 1-GHz bandwidth scopes in the 3000 X-Series, which were used for the measurement examples in the first part of this paper, exhibit this type of frequency response. These scopes sample at 5 GSa/s when only half of the channels are turned on or 2.5 GSa/s when all channels are turned on.

For a 1-GHz bandwidth scope that exhibits a Gaussian frequency response (red curve), the Nyquist frequency (f_{N2}) should be no lower than 2 GHz in order to minimize sampling frequency components above the Nyquist frequency. This requires a minimum sample rate of 4 GSa/s which results in a minimum sample rate-to-bandwidth ratio of 4:1. Agilent’s 1-GHz bandwidth scopes in the InfiniiVision 7000 Series exhibit this type of frequency response.

For either type of frequency response (maximally-flat or Gaussian), it is impossible to completely eliminate the possibility of sampling frequency components above the Nyquist frequency (red and green shaded areas of Figure 8). But with the guidelines presented here, the affects of sampling frequency components above the Nyquist frequency can be minimized. It should also be noted that if you are attempting to capture signals that contain frequency components at or near the Nyquist frequency of your oscilloscope (half the scope’s maximum sample rate), perhaps it is time to consider using a higher bandwidth oscilloscope.

So which type of frequency response is the “right” frequency response for an oscilloscope? There is no “right” answer. Each type of frequency response has its advantages. A scope with a maximally-flat frequency response captures in-band frequencies more accurately while eliminating more of the higher frequency components that can induce aliasing. In addition, minimum required sample rate is lower. This means that a scope with this type of response can be designed with lower cost components resulting in a lower priced scope for the end-user. Agilent’s new 1-GHz bandwidth oscilloscopes in the InfiniiVision 3000 X-Series are the lowest priced 1-GHz bandwidth scopes on the market today from any major scope vendor. Another advantage of sampling at a lower rate — assuming that the lower rate provides virtually the same level of detail as demonstrated in this paper — is that the scope can capture a longer time-span based on a fixed amount of the acquisition memory (Time-span captured = Memory Depth/Sample Rate).

Figure 8: Typical oscilloscope frequency responses.
The major advantage of scopes with a Gaussian frequency response is that they can capture out-of-band frequency components more accurately which results in a slightly faster oscilloscope rise time specification. But this requires a higher sample rate (minimum sample rate-to-bandwidth ratio of 4:1). But if you are attempting to make accurate measurements on signals that contain frequency components above the bandwidth of the oscilloscope, you really need a higher bandwidth scope anyway.

To learn more about this topic, refer to the application note listed at the end of this document titled, “Understanding Oscilloscope Frequency Response and its Effect on Rise Time Accuracy”.

Summary

As shown in this paper, a 1-GHz bandwidth scope with a minimum sample rate-to-bandwidth ratio of 2.5:1 can accurately capture both high frequency analog signals as well as fast-edge transitions on digital signals. Even though unfiltered sample density (400 ps sample spacing) may be low when viewing high frequency waveforms on fast timebase ranges, automatic \( \sin(x)/x \) waveform reconstruction increases interpolated sample density by a factor of up to 20 to provide an accurate and continuous waveform display while updating waveforms as fast as 1,000,000 waveforms/sec.

Note that although the measurement examples presented in this paper primarily focused on 1-GHz bandwidth oscilloscopes, the principles and theory can be applied to any bandwidth oscilloscope.


\[ 2 \text{ An oscilloscope’s rise time specification is a theoretical characteristic of a scope based on inputting an infinitely fast transition (0.0 ps), which is impossible to test. Scopes that exhibit a Gaussian frequency response typically have a calculated rise time specification (based on 10\% to 90\% threshold criteria) equal to } 0.35/f_{BW} \text{. Scopes that exhibit a maximally-flat frequency response typically have a calculated rise time specification equal to } 0.45/f_{BW}. \]

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